

# Vector Spaces

## *Overview of vector spaces, subspaces, and linear (in)dependence*

### Objectives

To better understand vector spaces, subspaces, linear (in)dependence of vectors, basis of a vector space.

### Recitation Worksheet Problems: Sections 5.1, 5.2, 5.3

1. Sketch a graph of the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -3 \\ 1 \end{bmatrix},$$

- Does the linear system  $\mathbf{v}_1 x_1 + \mathbf{v}_2 x_2 = \mathbf{b}$  have a solution? Determine if the solution is unique.
- Write the above system in the form of two (scalar) equations and interpret the existence (or not) of a solution in terms of lines intersecting (or not).
- Write the above system in matrix form. Determine if the matrix is invertible. If so, find the solution using the formula for matrix inverse.

2. Consider the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -3 \\ 9 \end{bmatrix},$$

and given a real number  $h$ , define  $\mathbf{c} = \begin{bmatrix} 2 \\ h \end{bmatrix}$ . Plot the vectors  $\mathbf{v}_1$  and  $\mathbf{v}_2$  and find all values of  $h$  such that the system  $\mathbf{v}_1 x_1 + \mathbf{v}_2 x_2 = \mathbf{c}$  has a solution. Determine if the solution is unique.

3. Determine which of the sets of vectors below is linearly independent, and also answer the following:

- If the set is linearly dependent, express one vector as a non-zero linear combination of the other vectors in the set.
- If the set is linearly independent, show that the only linear combination of the above vectors which gives the zero vector is such that all scalars are zero.
- For each of the sets, determine if the span of the vectors is the whole space, a plane, or a line.
- For each of the sets, find a basis for their span.

$$\text{(A)} \quad \left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\};$$

$$\text{(B)} \quad \left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \right\};$$

$$\text{(C)} \quad \left\{ \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ -6 \\ 0 \end{bmatrix} \right\};$$

$$\text{(D)} \quad \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \right\};$$

$$\text{(E)} \quad \left\{ \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 6 \\ -4 \end{bmatrix}, \begin{bmatrix} 3 \\ -9 \\ 6 \end{bmatrix} \right\}.$$