Second Order Equations

Solving second order linear nonhomogeneous differential equations

Objectives

Students should be able to do the following:

- To solve second order, linear, nonhomogeneous differential equations with simple sources.
- To compute the Laplace Transform of simple functions.
- To use the Laplace Transform to solve second order, linear differential equations.

Recitation Worksheet Problems: Sections 2.3, 3.1, 3.2

1. Use the **undetermined coefficients method** to find the general solution of the ODE

$$y'' + 5y' + 6y = 3\sin(2t).$$

2. Consider the following IVP

$$y'' + 5y' + 6y = e^{-3t}, \quad y(0) = 0, \quad y'(0) = 0.$$

(a) Use the **undetermined coefficients method** to find the solution of the above IVP.

(b) Use the **Laplace Transform Method** to find the solution of the above IVP.

Note: To use the Laplace Transform Method, expand your solution using partial fractions, but in the interest of time, you do not need to find the exact values of the constants in this expansion. Your final solution for $y(t)$ could involve general constants A, B, and C.

Answers

1. $y(t) = c_1\, e^{-2t} + c_2\, e^{-3t} + \dfrac{3}{52}\,\sin(2t) - \dfrac{15}{52}\,\cos(2t)$.

2. (a) $y(t) = e^{-2t} - e^{-3t} - t\, e^{-3t}$.

(b) If $\dfrac{1}{(s+2)(s+3)^2} = \dfrac{A}{(s+2)} + \dfrac{B}{(s+3)} + \dfrac{C}{(s+3)^2}$, the the solution of the IVP is

$$y(t) = A\, e^{-2t} + B\, e^{-3t} + C\, t\, e^{-3t}.$$

If time is available, one can find $A = 1$, $B = -1$, and $C = -1$, to arrive at $y(t) = e^{-2t} - e^{-3t} - t\, e^{-3t}$.