

Introduction to Modeling

Construction of Models and Description of their Solutions

Objectives

- Students should be able to create mathematical models from a given description of population systems.
- Conversely, given the differential equations of a population model, students should know the meaning of the equation coefficients and then be able to describe what type of population system is described by these equations.
- Students should be able to obtain a qualitative description of the solutions of certain differential equations.

Recitation Worksheet Problems: Sections 1.1, 1.2, 1.3, 1.4

1. A radioactive material has a **decay rate** proportional to the amount of radioactive material present at that time, with a proportionality factor of 5 per unit time.
 - (a) Write a differential equation of the form $P' = F(P)$, which models this situation, where P is the amount of radioactive material micrograms as function of time.
 - (b) Now, assume that we have the same radioactive material decaying as above, but we are **adding** additional material (of the same type) **at a constant rate of 3** micrograms per unit time. Write the differential equation in this case.
 - (c) Solve the ordinary differential equation in part (b) above, assuming the initial amount of radioactive material is 5 micrograms.

2. Consider the population model

$$\frac{dP}{dt} = -3(P - 5)(P - 10)(P - 20),$$

where $P(t)$ is the population at time t .

- (a) Determine the **equilibria** of the differential equation.
- (b) Determine the intervals where the population is **increasing** and where it is **decreasing**.
- (c) Determine the stability of the equilibria.

3. Consider two populations of organism, which occupy the same environment. Let $A(t)$ and $B(t)$ denote the number of organisms at time t of the first and second kind, respectively. They are modeled by the following system of equations

$$\begin{aligned}A' &= 2A - \frac{A^2}{10} - 3AB \\B' &= 3B - \frac{B^2}{5} + 2AB.\end{aligned}$$

- Determine the growth rate coefficients of A and B (when the organisms are not feeling that resources are limited).
 - Determine the carrying capacity of each of the populations (if the other population didn't exist).
 - Determine the interaction between the two populations: are they cooperating, are they competing? If they have different types of behaviors, which one is helping and which one is hurting the other?
4. Consider the initial value problem

$$y'(t) = \frac{t}{y(t) + t^2 y(t)}, \quad y(0) = 1.$$

- Find an **implicit** expression of all solutions y of the differential equation above, in the form $\psi(t, y) = c$, where c collects all constant terms.
- Find the **explicit** expression of the solution y of the initial value problem above.