# Introduction to Modeling

*Construction of Models and Description of their Solutions*

## Objectives

- Students should be able to create mathematical models from a given description of a physical situation.
- Conversely, given the differential equations (for example, of a population model), students should know the meaning of the equation coefficients and then be able to describe what type of population system is described by these equations.
- Students should be able to obtain a qualitative description of the solutions of autonomous differential equations.

## Recitation Worksheet Problems: Sections 1.2, 1.3

**(1)** A population of fish has a **growth rate proportional to the amount of fish present** at that time, with a proportionality factor of $\frac{1}{3}$ per unit time.

**(1a)** Write a differential equation of the form $P' = F(P)$, which models this situation, where P is the number of fish as a function of time.

**(1b)** Now, assume that we have the same fish population, reproducing as above, but we are harvesting fish at a **constant rate of 300** fish per unit time. Write the differential equation in this case.

**(1c)** Assume that the initial fish population is 500 fish. Solve the ordinary differential equation in part (1b) above with this given initial condition.

**(2)** A radioactive material has a **decay rate** proportional to the amount of radioactive material present at that time, with a proportionality factor of 5 per unit time.

**(2a)** Write a differential equation of the form $P' = F(P)$, which models this situation, where P is the amount of radioactive material micrograms as function of time.

**(2b)** Now, assume that we have the same radioactive material decaying as above, but we are **adding** additional material (of the same type) **at a constant rate of 3** micrograms per unit time. Write the differential equation in this case.

**(2c)** Solve the ordinary differential equation in part (2b) above, assuming the initial amount of radioactive material is 10 micrograms.

**(3)** Consider the population model

$$\frac{dP}{dt} = -3\,(P-5)\,(P-10)\,(P-20)\,,$$

where $P(t)$ is the population at time $t$.

**(3a)** Determine the **equilibria** of the differential equation.

**(3b)** Determine the intervals where the population is **increasing** and where it is **decreasing**.

**(3c)** Determine the stability of the equilibria.

**(4)** Consider two populations of organisms, which occupy the same environment. Let $A(t)$ and $B(t)$ denote the number of organisms at time $t$ of the first and second kind, respectively. They are modeled by the following system of equations

$$A' = 2A - \frac{A^2}{10} - 3AB$$
$$B' = 3B - \frac{B^2}{5} + 2AB.$$

**(4a)** Determine the growth rate coefficients of $A$ and $B$ (when the organisms are not sensing that resources are limited).

**(4b)** Determine the carrying capacity of each of the populations (if the other population didn't exist).

**(4c)** Determine the interaction between the two populations: are they cooperating, are they competing? If they have different types of behaviors, which one is helping and which one is hurting the other?