

# Linear Systems of Differential Equations

## *Two-Dimensional linear systems of differential equations*

### Objectives

To understand the solutions of  $2 \times 2$  linear systems of first order differential equations having constant coefficients and no sources.

### Recitation Worksheet Problems: Sections 6.1, 6.2

Consider the matrices

$$(a) \quad A = \begin{bmatrix} -3 & 4 \\ -1 & -3 \end{bmatrix}, \quad (b) \quad A = \begin{bmatrix} 3 & 4 \\ 1 & 0 \end{bmatrix}, \quad (c) \quad A = \begin{bmatrix} 7 & -2 \\ -4 & 5 \end{bmatrix}, \quad (d) \quad A = \begin{bmatrix} -2 & -1 \\ 1 & -4 \end{bmatrix}.$$

For each of the matrices **(a)**-**(d)** above do the following:

- (1) Find the eigenvalues and eigenvectors of the matrices  $A$ .
- (2) Classify the trivial solution  $\mathbf{x} = \mathbf{0}$  as one of the following: Source Node, Sink Node, Saddle Node, Source Spiral, Sink Spiral, Center.
- (3) Find a set of real fundamental solutions of the system  $\mathbf{x}' = A\mathbf{x}$ .
- (4) Find the particular solution satisfying  $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ .
- (5) Only for cases **(a)**-**(c)** sketch a phase portrait of the solutions of the system  $\mathbf{x}' = A\mathbf{x}$ .

**Note:** Make sure your phase portrait captures all qualitatively different solutions.