4.2. The Initial Value Problem

Section Objective(s):
- Overview of the LT Method.
- Homogeneous IVP.
- Non-Homogeneous IVP.
- Higher Order IVP.

4.2.1. Overview of the LT Method.

Overview: The Laplace transform (LT) can be used to solve differential equations:

$$\mathcal{L}\begin{bmatrix}\text{differential eq.}\\ \text{for } y(t).\end{bmatrix} \xrightarrow{(1)} \begin{matrix}\text{Algebraic eq.}\\ \text{for } \mathcal{L}[y(t)].\end{matrix} \xrightarrow{(2)} \begin{matrix}\text{Solve the}\\ \text{algebraic eq.}\\ \text{for } \mathcal{L}[y(t)].\end{matrix} \xrightarrow{(3)} \begin{matrix}\text{Transform back}\\ \text{to obtain } y(t).\\ \text{(Use the table.)}\end{matrix}$$

When the LT Method works:

(a) The LT method works with ______________________________ equations only.

(b) The LT method works with ____________________ sources

and ____________________ sources.

Why the LT Method works: Because it satisfies the One-to-One Property.

The One-to-One Property: When we Laplace transform the differential equation and solve for $\mathcal{L}[y]$ we get an expression of the form

______________________________,

We then used a Laplace transform table to find a function _______ such that

______________________________.

So we arrive to an equation of the form

______________________________.

Does the expression above imply that $y(t) = h(t)$? The answer is __________.

Theorem 4.2.1. (One-to-One) If f, g are continuous on $[0, \infty)$ and bounded by an exponential, then

__________________________ $\Rightarrow$ __________________.

4.2.2. **Homogeneous IVP.**

EXAMPLE 4.2.2: Use the Laplace transform to find the solution y to the initial value problem

$$y'' - y' - 2y = 0, \qquad y(0) = 1, \qquad y'(0) = 0.$$

SOLUTION:

4.2.3. Non-Homogeneous IVP.

EXAMPLE 4.2.4: Use the Laplace transform to find the solution y to the initial value problem

$$y'' - 4y' + 4y = 3\,e^{t}, \qquad y(0) = 0, \qquad y'(0) = 0.$$

SOLUTION:

$\triangleleft$

4.2.4. **Higher Order IVP.**

Example 4.2.6: Use the Laplace transform to find the solution y to the initial value problem

$$y^{(4)} - 4y = 0, \qquad \begin{aligned} y(0) &= 1, & y'(0) &= 0, \\ y''(0) &= -2, & y'''(0) &= 0. \end{aligned}$$

Solution:

$\lhd$