2.6. Applications

Section Objective(s):

- Review and Overview of Names Used in Physics.
- Undamped Mechanical Oscillations.
- Damped Mechanical Oscillations.

2.6.1. Review and Overview of Names Used in Physics.

Review: To find fundamental solutions to constant coefficient homogeneous equations

$$y'' + a_1\, y' + a_0\, y = 0, \qquad a_1, a_2 \in \mathbb{R}. \tag{2.6.1}$$

one needs to find the roots or the characteristic polynomial $p(r) = r^2 + a_1 r + a_0$, which are

____.

We then have three different cases to consider.

(a) A system is ____________________________ iff $r_\pm \in \mathbb{R}$ and $r_- < r_+ < 0$. A set of fundamental solutions is formed by the decreasing exponentials,

____.

(b) A system is ____________________________ iff $r_\pm \in \mathbb{R}$ and $r_- = r_+ = r_0 < 0$. A set of fundamental solutions is

____.

(c) A system is ______________________________ iff $r_\pm = \alpha \pm i\beta \in \mathbb{C}$ and $\alpha < 0$. A set of fundamental solutions is

____.

(d) A system is ______________________ iff $r_\pm = \alpha \pm i\beta \in \mathbb{C}$ and $\alpha = 0$. A set of fundamental solutions is

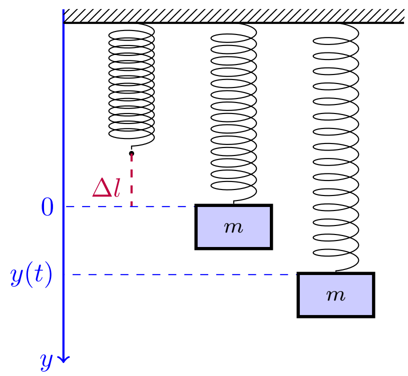
____.

2.6.2. **Undamped mechanical oscillations.**

Problem Describe the movement of a body attached to a spring oscillating in a region where the spring does not deform in a permanent way.

Definition 2.6.1. A ____________________ is an object that when deformed by an amount Δl creates a force ____________________, with $k > 0$.

Remark: The negative sign in the spring force means that force F_s and the displacement Δl are on ______________________________.



Theorem 2.6.2. (Static Equilibrium) A spring with spring constant k, an attached body mass m, at rest with a spring deformation Δl, satisfies

______________________________.

Proof of Theorem 2.6.2:

Remark: It is possible to compute the spring constant k by measuring the displacement Δl and knowing the body mass m.

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Theorem 2.6.3. (Movement without Drag) The vertical movement of a spring and a body in the air with spring constant $k > 0$ and body mass $m > 0$ is described by

____,

where y is the vertical displacement function. Furthermore, there is a unique solution to equation above satisfying the initial conditions $y(0) = y_0$ and $y'(0) = v_0$,

____,

with angular ______________________________,

where the ____________________ and _______________

_______________, are fixed by the initial conditions $y(0) = y_0$ and $y'(0) = v_0$,

____.

Remark:

Proof of Theorem 2.6.3:

□

Example 2.6.1: Find the movement of a 50 gr mass attached to a spring moving in air with initial conditions $y(0) = 4\,\mathrm{cm}$ and $y'(0) = 40\,\mathrm{cm/s}$. The spring is such that a 30 gr mass stretches it 6 cm. Approximate the acceleration of gravity by $1000\,\mathrm{cm/s^2}$.

Solution:

2.6.3. **Damped Mechanical Oscillations.**

Remarks:

(a) Damping is caused by ________________________________.

(b) We study ____, where $d > 0$.

____.

(c) Example: A spring oscillating inside an oil bath.

Theorem 2.6.4. (Movement with Drag)

(a) The vertical displacement y of a spring and a body with spring constant $k > 0$, body mass $m > 0$, and damping constant $d \geqslant 0$, is described by the solutions of

____ (2.6.2)

(b) The roots of the characteristic polynomial of Eq. (2.6.2) are

____

with damping coefficient $\omega_d = \dfrac{d}{2m}$ and circular frequency $\omega_0 = \sqrt{\dfrac{k}{m}}$.

(c) The solutions to Eq. (2.6.2) fall into one of the following cases:

(i) A system with $\omega_d > \omega_0$ is over damped, with general solution to Eq. (2.6.2)

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(ii) A system with $\omega_d = \omega_0$ is critically damped, with general solution to Eq. (2.6.2)

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(iii) A system with $\omega_d < \omega_0$ is under damped, with general solution to Eq. (2.6.2)

____

where ____________________________.

Remark: In the case the damping coefficient vanishes we recover Theorem ???.

Proof of Therorem 2.6.4:

□

Example 2.6.2: Find the movement of a 5kg mass attached to a spring with constant $k = 5$ kg/s^2 moving in a medium with damping constant $d = 5$ kg/s, with initial conditions $y(0) = \sqrt{3}$ and $y'(0) = 0$.

Solution:

ϕ	$\sin \phi$	$\cos \phi$	$\tan(\phi)$
0	0	1	0
$\pi/6$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\pi/2$	1	0	∞