2.4. Euler Equidimensional Equation

Section Objective(s):

- The Main Result and the Indicial Equation.
- Proving the Repeated Root Case.
- Real Solutions for Complex Roots.

2.4.1. **The Roots of the Indicial Polynomial.**

Definition 2.4.1. The ***Euler equidimensional equation*** at $t_0 \in \mathbb{R}$ is

____.

Remark: If $t_0 = 0$, the equation is

____.

EXAMPLE 2.4.1: Find the general solution of the equation below, for $t > 0$,

$$t^2\, y'' + 4t\, y' + 2\, y = 0.$$

SOLUTION:

◁

Theorem 2.4.2. (Euler Equation) Consider the Euler equidimensional equation

$$\underline{\hspace{8cm}}, \qquad (2.4.1)$$

where a_1, a_0, and t_0 are real constants, and denote by $r_\pm$ the roots of the indicial polynomial $p(r) = r(r-1) + a_1 r + a_0$.

(a) If ____________, real or complex, then the general solution of Eq. (2.4.1) is

____.

(b) If ____________________, real, then the general solution of Eq. (2.4.1) is

____.

Remark:

Proof of Theorem 2.4.2:

□

EXAMPLE 2.4.2: Find the general solution of the Euler equation below for $t > 0$,

$$t^2\, y'' - 3t\, y' + 4\, y = 0.$$

SOLUTION:

$\lhd$

EXAMPLE 2.4.3: Find the general solution of the Euler equation below for $t > 0$,

$$t^2\, y'' - 3t\, y' + 13\, y = 0.$$

SOLUTION:

$\lhd$

2.4.2. **Real Solutions for Complex Roots.**

Theorem 2.4.3. (Real Valued Fundamental Solutions) If the differential equation

____,

where a_1, a_0, t_0 are real constants, has indicial polynomial with complex roots

____________________ and complex valued fundamental solutions for $t > t_0$,

____,

then the equation also has real valued fundamental solutions for $t > t_0$ given by

____.

Proof of Theorem 2.4.3:

Example 2.4.4: Find a real-valued general solution of the Euler equation below for $t > 0$,

$$t^2\, y'' - 3t\, y' + 13\, y = 0.$$

Solution:

$\triangleleft$