2.3. Homogenous Constant Coefficients Equations

Section Objective(s):

- The Characteristic Equation.
- Proving the Repeated Root Case.
- Real Solutions for Complex Roots.

2.3.1. **The Characteristic Equation.**

Remark: Recall the General Solution Theorem:

- If we find ____________________ solutions y_1, y_2 of

  ____,

  then we know all solutions ______________________________.

- It is simple to find ____________________ solutions in the case of constant coefficient equations.

Example 2.3.1: Find all solutions to the equation $y'' + 5y' + 6y = 0$.

Solution:

◁

Definition 2.3.1. The ***characteristic polynomial*** and ***characteristic equation*** of the differential equation

____,

are, respectively,

____.

Theorem 2.3.2. If $r_{\pm}$ are the roots of the characteristic polynomial of

$$\underline{\hspace{10cm}}, \qquad (2.3.1)$$

and if c_{+}, c_{-} are arbitrary constants, then we have the following:

(a) If ________________, real or complex, then the general solution of Eq. (2.3.1) is

$$\underline{\hspace{10cm}}.$$

(b) If ______________________, real, then the general solution of Eq. (2.3.1) is

$$\underline{\hspace{10cm}}.$$

Proof of Theorem 2.3.2:

□

Example Similar to 2.3.3: Find the solution y of the initial value problem

$$y'' - y' - 2y = 0, \qquad y(0) = 1, \qquad y'(0) = 5.$$

Solution:

$\triangleleft$

Example Similar to 2.3.4: Find the general solution of $y'' + 6y' + 9y = 0$.

Solution:

◁

Example 2.3.5: Find the general solution y_{gen} of the equation

$$y'' - 2y' + 6y = 0.$$

Solution:

◁

2.3.2. **Real Solutions for Complex Roots.**

Review of Complex Numbers:

- Complex numbers have the form ________________, where ______________.
- The complex conjugate of z is the number ________________.
- $\text{Re}(z) = a$, $\text{Im}(z) = b$ are the real and imaginary parts of z
- Hence: ______________________________, ______________________________.
- The exponential of a complex number is defined as

  ____.

  In particular holds __________________________________.
- Euler's formula: ____.
- Hence, a complex number of the form e^{a+ib} can be written as

____.

- From e^{a+ib} and e^{a-ib} we get the real numbers

____.

Theorem 2.3.3. (Real Valued Fundamental Solutions) If the equation

____

has coefficients such that ________________________, and we denote the roots of $p(r) = r^2 + a_1 r + a_0$ as

____,

then, complex valued fundamental solutions of the differential equation are

____,

while real valued fundamental solutions of the differential equation are

____.

Proof of Theorem 2.3.3:

EXAMPLE 2.3.7: Find the real valued general solution of the equation

$$y'' - 2y' + 6y = 0.$$

SOLUTION:

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