1.3. Separable Equations

Section Objective(s):

- Separable Differential Equations
- Euler Homogeneous Equations
- Solving Euler Homogeneous Equations

1.3.1. Separable Differential Equations.

Definition 1.3.1. A ***separable*** differential equation for the function y is

$$\rule{8cm}{0.4pt},$$

where $\rule{1.5cm}{0.4pt}$ are given functions.

Remark:

Example 1.3.1:

(a) $\left(1-y^2\right) y' = t^2$.

(b) $\dfrac{1}{y^2}\, y' = -\cos(2t)$.

(c) $y' = a(t)\, y$.

(d) $y' = e^y + \cos(t)$.

(e) $y' = a(t)\, y + b(t)$.

$\triangleleft$

EXAMPLE 1.3.2: Find all solutions y to the differential equation

$$-\frac{y'}{y^2} = \cos(2t).$$

SOLUTION:

$\triangleleft$

Theorem 1.3.2. (Separable Equations) If h, g are continuous, with $h \neq 0$, then

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has infinitely many solutions y satisfying the algebraic equation

____,

where ______________________ are antiderivatives of ____________________.

Remark:

Proof of Theorem 1.3.2:

$\square$

Example 1.3.3: Find all solutions y to the differential equation

$$y' = \frac{t^2}{1 - y^2}.$$

Solution:

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1.3.2. **Euler Homogeneous Equations.**

Definition 1.3.4. An ***Euler homogeneous*** differential equation has the form

____.

Remark:

(a) Any function F of t, y that depends only on the quotient y/t is

________________________________. This means that F does not change when we do the transformation ________________________________,

____.

For this reason the differential equations above are also called

________________________________ equations.

(b) Scale invariant functions are a particular case of

____,

which are functions f satisfying

____.

________________________________ functions are the case ________________.

Theorem 1.3.5. If the functions N, M, of t, y, are homogeneous of the same degree, then the differential equation

is Euler homogeneous.

Proof of Theorem 1.3.5:

$\square$

EXAMPLE 1.3.11: Show that $(t-y)\,y' - 2y + 3t + \dfrac{y^2}{t} = 0$ is an Euler homogeneous equation.

SOLUTION:

$\triangleleft$

1.3.3. **Solving Euler Homogeneous Equations.**

Theorem 1.3.6. The Euler homogeneous equation

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for the function y determines a separable equation for ________________, given by

_______________________________________.

Proof of Theorem 1.3.6:

$\square$

EXAMPLE 1.3.13: Find all solutions y of the differential equation $y' = \dfrac{t^2 + 3y^2}{2ty}$.

SOLUTION:

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