Name: ______________________ **PID:** ______________________

Section: ____ **Recitation Instructor**: ______________________

READ THE FOLLOWING INSTRUCTIONS.

- **Do not open your exam until told to do so.**
- Without fully opening the exam, check that you have pages 1 through 16.
- Fill in your name, etc. on this first page.
- In the **Multiple Choice** problems **write your answers in the table of page 2**.
- In the **Show Your Work** problems, **problems 10, 11, and 12**, you must show all your work. Write your answers clearly. Include enough steps for you to find possible mistakes when you revise your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- If you need scratch paper, use the back of the previous page.
- First do the problems you know how to do. Do not spend too much time on any particular problem. Return to the difficult problems later.
- If you have any questions please raise your hand and a proctor will come to you.
- You will be given exactly **90 minutes** for this exam.

ACADEMIC DISHONESTY.

- **No calculators, no phones, or any other electronic devices can be used on this exam**.
- Clear your desk of everything excepts pens, pencils and erasers.
- There is **no talking** allowed during the exam. Please **do not look at other students papers**.
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe. All cases of academic dishonesty will be reported immediately to the Dean of Undergraduate Studies and added to the student's academic record.

I have read and understand the above instructions: ______________________

SIGNATURE

Answers to Multiple Choice Questions.

Students must **write their answers** to the Multiple Choice questions in the table below.

Instructors **will not look** at the pages with the Multiple Choice questions.

Instructors **will look only at this table** to grade your answers to the Multiple Choice questions.

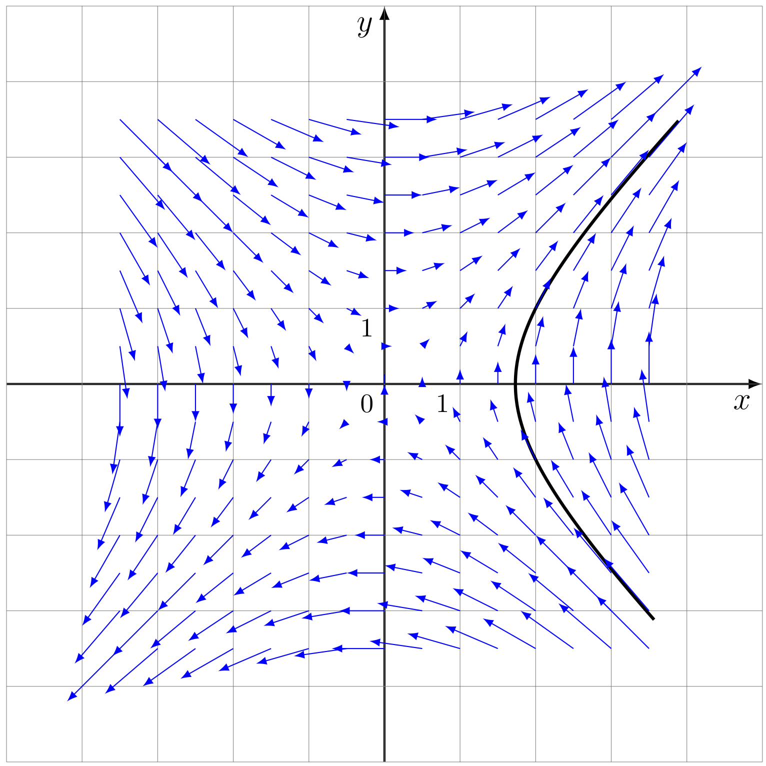
Question	Points	Answer Letter	Score
1	5		
2	5		
3	5		
4	5		
5	5		
6	5		
7	5		
8	5		
9	5		
Total	45		

1. (5 points) **Question 1.**

   Find the eigenpairs of matrix A and the vector $\boldsymbol{x}_0$ such that the **initial value problem**

$$\boldsymbol{x}'(t) = A\boldsymbol{x}(t), \qquad \boldsymbol{x}(0) = \boldsymbol{x}_0,$$

   has the solution curve displayed in the phase portrait.



(A) $\lambda_1 = 1, \quad \lambda_2 = -1,$

$$\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \boldsymbol{v}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad \boldsymbol{x}_0 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}.$$

(B) $\lambda_1 = 1, \quad \lambda_2 = -1,$

$$\boldsymbol{v}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad \boldsymbol{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \boldsymbol{x}_0 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$

(C) $\lambda_1 = 2, \quad \lambda_2 = 1,$

$$\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \boldsymbol{v}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad \boldsymbol{x}_0 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$

(D) $\lambda_1 = 1, \quad \lambda_2 = -1,$

$$\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \boldsymbol{v}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad \boldsymbol{x}_0 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

(E) $\lambda_1 = 1, \quad \lambda_2 = -1,$

$$\boldsymbol{v}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad \boldsymbol{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \boldsymbol{x}_0 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

(F) None of the above.

Important: Circle only **one** option.

Answer: **(D)**

2. (5 points) **Question 2.**

Find the inverse of the matrix $P = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

(A) $P^{-1} = -\frac{1}{2}\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$

(B) $P^{-1} = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

(C) $P^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$

(D) $P^{-1} = \begin{bmatrix} -4 & 2 \\ 3 & -1 \end{bmatrix}$

(E) $P^{-1} = -2\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

(F) $P^{-1} = \frac{1}{2}\begin{bmatrix} -4 & 2 \\ 3 & -1 \end{bmatrix}$

(G) None of the above.

Important: Circle only **one** option.

Answer: **(F)**

3. (5 points) **Question 3.**

Find the matrix e^{At}, where $t \in \mathbb{R}$ and A is the matrix with eigenpairs

$$\lambda_1 = -2, \quad \boldsymbol{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad \text{and} \quad \lambda_2 = 3, \quad \boldsymbol{v}_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

(A) $e^{At} = \frac{1}{5} \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$

(B) $e^{At} = \frac{1}{5} \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} e^{3t} & 0 \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$

(C) $e^{At} = -\frac{1}{5} \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{3t} \end{bmatrix} \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$

(D) $e^{At} = \frac{1}{5} \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{3t} \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$

(E) $e^{At} = \frac{1}{5} \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} e^{3t} & 0 \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 3 & -1 \end{bmatrix}$

(F) $e^{At} = \frac{1}{5} \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} -2t & 0 \\ 0 & 3t \end{bmatrix} \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$

(G) None of the above.

Important: Circle only **one** option.

Answer: **(E)**

4. (5 points) **Question 4.**

Which of the following sets are subspaces of $\mathbb{R}^3$?

(A) Any plane containing the origin.

(B) The intersection of two planes, when each plane contain the origin.

(C) Any line.

(D) Any plane parallel to the xz-plane.

(E) Spheres with center at the origin.

(F) Any line containing the origin.

(G) $\mathbb{R}^3$.

(H) All of the above.

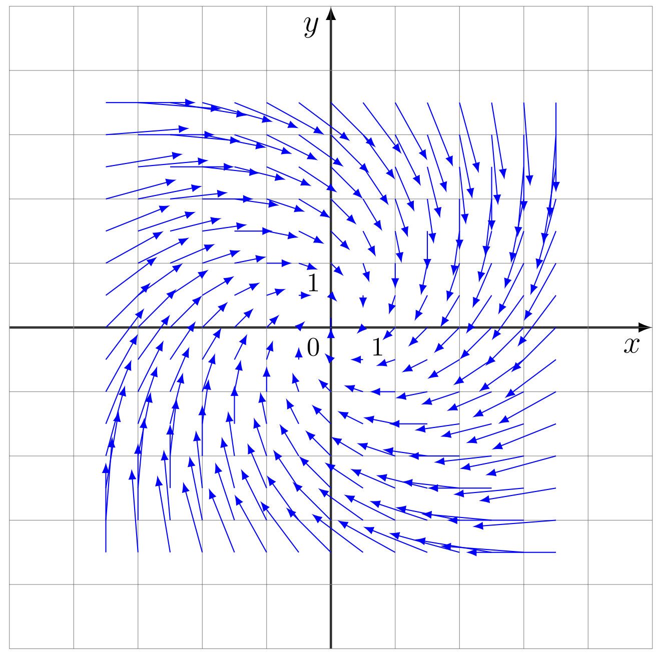
(I) None of the above.

Important: Circle **all** that apply. No grade for partially correct answers

(A), **(B)**, **(F)**, **(G)**.

5. (5 points) **Question 5.**

Find the stability of the solution $\boldsymbol{x} = \mathbf{0}$ of the linear system $\boldsymbol{x}'(t) = A\boldsymbol{x}(t)$ having the direction field given in the picture.



(A) Stable Node.

(B) Unstable Node.

(C) Saddle Node.

(D) Stable Spiral.

(E) Unstable Spiral.

(F) Center.

(G) None of the above.

Important: Circle only **one** option.

Answer: **(D)**

6. (5 points) **Question 6.**

Find the stability of the critical point $(1, 0)$ for the nonlinear system

$$\boldsymbol{x}' = \boldsymbol{f}(\boldsymbol{x}), \quad \text{where} \quad \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} x_1(1 - x_1) - x_1 x_2 \\ x_2\left(\frac{3}{4} - x_2\right) - \frac{1}{2} x_1 x_2 \end{bmatrix}.$$

(A) Stable Node.

(B) Unstable Node.

(C) Saddle Node.

(D) Stable Spiral.

(E) Unstable Spiral.

(F) Center.

(G) None of the above.

Important: Circle only **one** option.

Answer: **(C)**

7. (5 points) **Question 7.**

Find the eigenpairs of the matrix $A = \begin{bmatrix} 5 & -12 \\ 2 & -5 \end{bmatrix}$.

(A) $\lambda_1 = 1, \quad \boldsymbol{v}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\lambda_2 = 2, \quad \boldsymbol{v}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

(B) $\lambda_1 = -7, \quad \boldsymbol{v}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\lambda_2 = 7, \quad \boldsymbol{v}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

(C) $\lambda_1 = 1, \quad \boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\lambda_2 = -1, \quad \boldsymbol{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

(D) $\lambda_1 = 7, \quad \boldsymbol{v}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\lambda_2 = -7, \quad \boldsymbol{v}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

(E) $\lambda_1 = 1, \quad \boldsymbol{v}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\lambda_2 = -1, \quad \boldsymbol{v}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

(F) $\lambda_1 = 1, \quad \boldsymbol{v}_1 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ and $\lambda_2 = -1, \quad \boldsymbol{v}_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$.

(G) None of the above.

Important: Circle only **one** option.

Answer: **(E)**

8. (5 points) **Question 8.**

Find the constant k so that the function $\boldsymbol{x}(t) = \begin{bmatrix} k \\ 1 \end{bmatrix} e^{-2t}$ is a solution of

$$\boldsymbol{x}'(t) = \begin{bmatrix} -5 & 6 \\ -3 & 4 \end{bmatrix} \boldsymbol{x}(t).$$

(A) $k = -1$.

(B) $k = -2$.

(C) $k = -3$.

(D) $k = -4$.

(E) $k = 1$.

(F) $k = 2$.

(G) $k = 3$.

(H) $k = 4$.

(I) None of the above.

Important: Circle only **one** option.

Answer: **(F)**

9. (5 points) **Question 9.**

Find the stability of the critical point $\left(\frac{3}{2}, \frac{2}{3}\right)$ for the nonlinear system

$$\boldsymbol{x}' = \boldsymbol{f}(\boldsymbol{x}), \quad \text{where} \quad \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} 2x_1 - 3x_1x_2 \\ -3x_2 + 2x_1x_2 \end{bmatrix}.$$

(A) Stable Node.

(B) Unstable Node.

(C) Saddle Node.

(D) Stable Spiral.

(E) Unstable Spiral.

(F) Center.

(G) None of the above.

Important: Circle only **one** option.

Answer: **(F)**

10. (20 points) **Question 10.**

Consider a system of tortoises and rabbits, competing for the same finite food resources (grass for example). Assume that:

- r_t and r_r are the growth rate coefficients for tortoises and rabbits, respectively,
- K_t and K_r are the carrying capacity of tortoises and rabbits, respectively,
- α is the coefficient that measures how rabbits affect the tortoises' rate of change,
- β is the coefficient that measures how tortoises affect the rabbits' rate of change.

(a) (*12 points*) Write a system of differential equations modeling the tortoise and rabbit populations, denoted as $T(t)$ and $R(t)$.

(b) (*8 points*) Based on the physical meaning of the equation coefficients, fill in the blanks below with $>$, or $\simeq$, or $=$, or $<$. **Explain your choice.**

$$r_t \,\_\_\_\, r_r, \qquad K_t \,\_\_\_\, K_r, \qquad \alpha \,\_\_\_\, \beta.$$

(a)

$$\boxed{\begin{aligned} T' &= r_t\, T\Big(1 - \frac{T}{K_t}\Big) - \alpha\, T\, R \\ R' &= r_r\, R\Big(1 - \frac{R}{K_r}\Big) - \beta\, T\, R. \end{aligned}}$$

(b)

$$\boxed{r_t < r_r, \qquad K_t \simeq K_r, \quad (\text{also ok } K_t > K_r), \qquad \alpha \simeq \beta, \quad (\text{also ok } \alpha > \beta)}.$$

- $r_t < r_r$ because rabbits reproduce faster than tortoises.
- $K_t \simeq K_r$ because rabbits and tortoises have the same size, then the same environment can support the same numbers of rabbits and tortoises.
  Note: Answer $K_t > K_r$ is also considered correct if the argument is that tortoises are cold blooded, so they need less food than rabbits, so the environment can support more tortoises than rabbits.
- $\alpha \simeq \beta$ because rabbits and tortoises have the same size, then they affect each other in a similar way.
  Note: Answer $\alpha > \beta$ is also considered correct if the argument is that tortoises are cold blooded, so they need less food than rabbits, so rabbits affect more the tortoises' rate of change than tortoises affect the rabbits' rate of change.

11. (20 points) **Question 11.** Consider the following vectors.

$$\boldsymbol{v}_1 = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}, \qquad \boldsymbol{v}_2 = \begin{bmatrix} 3 \\ 2 \\ 6 \end{bmatrix}, \qquad \boldsymbol{v}_3 = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}.$$

(a) (*8 points*) Determine whether $\boldsymbol{v}_1$, $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$ are linearly dependent or linearly independent. If they are linearly dependent, express one as a linear combination of the others. If they are linearly independent, show that the only linear combination of the above vectors, which gives the zero vector has all coefficients zero.

(b) (*6 points*) Based on your work in (a), determine if the span of $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$ is a line, a plane, or the whole space.

(c) (*6 points*) Use your answer in (b) to determine for what vectors $\boldsymbol{b} \in \mathbb{R}^3$ the system below has a solution. Explain.

$$+ \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} x_1 + \begin{bmatrix} 3 \\ 2 \\ 6 \end{bmatrix} x_2 + \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} x_3 = \boldsymbol{b}.$$

(a)

$$\begin{bmatrix} 3 \\ 2 \\ 6 \end{bmatrix} = 3 \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} + 2 \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} \quad \Rightarrow \quad 3 \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 3 \\ 2 \\ 6 \end{bmatrix} + 2 \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

so the vectors are **linearly dependent**.

(b) The span of these three vectors is the **a plane in** $\mathbb{R}^3$.

(c) The system above **has solution for every vector $\boldsymbol{b}$ on the plane spanned by** $\left\{ \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} \right\}$.

12. (20 points) **Question 12.** Consider the linear system $\boldsymbol{x}'(t) = A\,\boldsymbol{x}(t)$, where matrix A has eigenpairs

$$\lambda_+ = 3, \quad \boldsymbol{v}_+ = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad \text{and} \qquad \lambda_- = -1, \quad \boldsymbol{v}_- = \begin{bmatrix} -1 \\ 2 \end{bmatrix}.$$

(a) (*4 points*) Write the general solution of the differential equation above.

(b) (*8 points*) Sketch a phase portrait of the solutions to the linear system above.
Label your axes, use the same scale in both axes, and indicate the direction of increasing time for the solutions plotted.

(c) (*2 points*) Determine the stability of the trivial solution $\boldsymbol{x} = \boldsymbol{0}$.

(d) (*2 points*) Find the solution of $\boldsymbol{x}' = A\,\boldsymbol{x}$ that satisfies the initial condition $\boldsymbol{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.
Show your work.

(e) (*2 points*) In the phase plane done in (b) plot the solution curve found in (d).

(f) (*2 points*) For the solution found in (d) find the following limits:

$$\lim_{t\to\infty} \frac{\boldsymbol{x}(t)}{\|\boldsymbol{x}(t)\|}, \qquad \text{and} \qquad \lim_{t\to\infty} \|\boldsymbol{x}(t)\|.$$

Justify your answer.

(a)

$$\boxed{\boldsymbol{x}(t) = c_+\, e^{3t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_-\, e^{-t} \begin{bmatrix} -1 \\ 2 \end{bmatrix}}.$$

(b) Sorry, we do not have a picture.

(c) Saddle node.

(d)

$$c_+ \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_- \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} c_+ \\ c_- \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} c_+ \\ c_- \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$c_+ = \frac{3}{5}, \quad c_- = \frac{1}{5} \quad \Rightarrow \quad \boxed{\boldsymbol{x}(t) = \frac{3}{5}\, e^{3t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \frac{1}{5}\, e^{-t} \begin{bmatrix} -1 \\ 2 \end{bmatrix}}.$$

Congratulations you are now done with the exam!

Go back and check: - You copied your answers to MC questions in the MC table.

- Your solutions to problems 10, 11, and 12 are accurate and clear.

- Your answers to problems 10, 11, and 12 are BOXED.

When you are completely happy with your work please bring your exam to the front to be handed in.

Please have your MSU student ID ready so that is can be checked.

DO NOT WRITE BELOW THIS LINE.

Question	Points	Score
MC	45	
10	20	
11	20	
12	20	
Total	105	
Maximum	100	

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}, \quad n \neq -1; \quad \int \frac{1}{x}\,dx = \ln|x|$$
$$\int e^{ax}\,dx = \frac{e^{ax}}{a}, \qquad \int a^x\,dx = \frac{a^x}{\ln a}$$
$$\int \ln(ax)\,dx = x\,(\ln(ax) - 1)$$
$$\int x^n\,\ln(ax)\,dx = \frac{x^{(n+1)}}{(n+1)^2}\,\big[(n+1)\ln(ax) - 1\big]$$
$$\int x\,e^{ax}\,dx = \frac{e^{ax}}{a^2}\,(ax - 1)$$
$$\int x^2\,e^{ax}\,dx = \frac{e^{ax}}{a^3}\,(a^2x^2 - 2ax + 2)$$
$$\int \sin(ax)\,dx = -\frac{1}{a}\,\cos(ax)$$
$$\int \cos(ax)\,dx = \frac{1}{a}\,\sin(ax)$$
$$\int x\sin(ax)\,dx = -\frac{x}{a}\,\cos(ax) + \frac{1}{a^2}\,\sin(ax)$$
$$\int x\cos(ax)\,dx = \frac{x}{a}\,\sin(ax) + \frac{1}{a^2}\,\cos(ax)$$
$$\int e^{ax}\,\sin(bx)\,dx = \frac{e^{ax}}{a^2+b^2}\,\big[a\,\sin(bx) - b\,\cos(bx)\big]$$
$$\int e^{ax}\,\cos(bx)\,dx = \frac{e^{ax}}{a^2+b^2}\,\big[b\,\sin(bx) + a\,\cos(bx)\big]$$
$$\int \tan(ax)\,dx = \frac{1}{a}\,\ln|\sec(ax)|$$
$$\int \sec^2(ax)\,dx = \frac{1}{a}\,\tan(ax)$$
$$\int \sec(ax)\,dx = \frac{1}{a}\,\ln|\sec(ax) + \tan(ax)|$$
$$\int \csc(ax)\,dx = -\frac{1}{a}\,\ln|\csc(ax) + \cot(ax)|$$
$$\int \sec(ax)\tan(ax)\,dx = \frac{1}{a}\,\sec(ax)$$
$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\,\arctan\Big(\frac{x}{a}\Big)$$
$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\Big(\frac{x}{a}\Big)$$
$$\int \frac{a}{x\sqrt{x^2-a^2}}\,dx = \operatorname{arcsec}\Big(\frac{x}{a}\Big)$$
$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \cosh^{-1}\Big(\frac{x}{a}\Big) = \ln(x + \sqrt{x^2-a^2})$$
$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \sinh^{-1}\Big(\frac{x}{a}\Big) = \ln(x + \sqrt{x^2+a^2})$$
$$\sinh(x) = \frac{e^x - e^{-x}}{2}, \qquad \cosh(x) = \frac{e^x + e^{-x}}{2}$$

$f(t)$	$F(s) = \mathcal{L}[f(t)]$	D_F
$f(t) = 1$	$F(s) = \dfrac{1}{s}$	$s > 0$
$f(t) = e^{at}$	$F(s) = \dfrac{1}{(s-a)}$	$s > a$
$f(t) = t^n$	$F(s) = \dfrac{n!}{s^{(n+1)}}$	$s > 0$
$f(t) = \sin(at)$	$F(s) = \dfrac{a}{s^2+a^2}$	$s > 0$
$f(t) = \cos(at)$	$F(s) = \dfrac{s}{s^2+a^2}$	$s > 0$
$f(t) = \sinh(at)$	$F(s) = \dfrac{a}{s^2-a^2}$	$s > \|a\|$
$f(t) = \cosh(at)$	$F(s) = \dfrac{s}{s^2-a^2}$	$s > \|a\|$
$f(t) = t^n e^{at}$	$F(s) = \dfrac{n!}{(s-a)^{(n+1)}}$	$s > \max\{a, 0\}$
$f(t) = e^{at}\sin(bt)$	$F(s) = \dfrac{b}{(s-a)^2+b^2}$	$s > \max\{a, 0\}$
$f(t) = e^{at}\cos(bt)$	$F(s) = \dfrac{(s-a)}{(s-a)^2+b^2}$	$s > \max\{a, 0\}$
$f(t) = e^{at}\sinh(bt)$	$F(s) = \dfrac{b}{(s-a)^2-b^2}$	$s > \max\{a, \|b\|\}$
$f(t) = e^{at}\cosh(bt)$	$F(s) = \dfrac{(s-a)}{(s-a)^2-b^2}$	$s > \max\{a, \|b\|\}$
$u(t-c)$	$\dfrac{e^{-cs}}{s}$	$s > 0,\ c \geqslant 0$
$\delta(t-c)$	e^{-cs}	$s \in \mathbb{R},\ c \geqslant 0$
$u(t-c)\,f(t-c)$	$e^{-cs}\,F(s)$	$c \geqslant 0$
$e^{ct}\,f(t)$	$F(s-c)$	$c \in \mathbb{R}$
$f'(t)$	$s\,F(s) - f(0)$	
$f''(t)$	$s^2\,F(s) - s\,f(0) - f'(0)$	
$(-t)^n\,f(t)$	$F^{(n)}(s)$	