Name: _____ **PID:** _____

Section: ____ **Recitation Instructor**: _____

READ THE FOLLOWING INSTRUCTIONS.

- **Do not open your exam until told to do so.**
- Without fully opening the exam, check that you have pages 1 through 16.
- Fill in your name, etc. on this first page.
- In the **Multiple Choice** problems **write your answers in the table of page 2**.
- In the **Show Your Work** problems, **problems 10, 11, and 12**, you must show all your work. Write your answers clearly. Include enough steps for you to find possible mistakes when you revise your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- If you need scratch paper, use the back of the previous page.
- First do the problems you know how to do. Do not spend too much time on any particular problem. Return to the difficult problems later.
- If you have any questions please raise your hand and a proctor will come to you.
- You will be given exactly **90 minutes** for this exam.

ACADEMIC DISHONESTY.

- **No calculators, no phones, or any other electronic devices can be used on this exam**.
- Clear your desk of everything excepts pens, pencils and erasers.
- There is **no talking** allowed during the exam. Please **do not look at other students papers**.
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe. All cases of academic dishonesty will be reported immediately to the Dean of Undergraduate Studies and added to the student's academic record.

I have read and understand the above instructions: _____

SIGNATURE

Answers to Multiple Choice Questions.

Students must **write their answers** to the Multiple Choice questions in the table below.

Instructors **will not look** at the pages with the Multiple Choice questions.

Instructors **will look only at this table** to grade your answers to the Multiple Choice questions.

Students Answers to Multiple Choice Questions

Question	Points	**Student Answers Here**	Score (for TA only)
1	5	**(D)**	
2	5	**(D)**	
3	5	**(G)**	
4	5	**(D)**	
5	5	**(D)**	
6	5	**(E)**	
7	5	**(E)**	
8	5	**(B) (E)**	
9	5	**(D) (G)**	
Total	45	Do not write in this box	

1. (5 points) **Question 1.**

   Identify the **differential equation** that produces the slope field below.

(A) $y' = y\,(y-2)$

(B) $y' = y\,(2-y)$

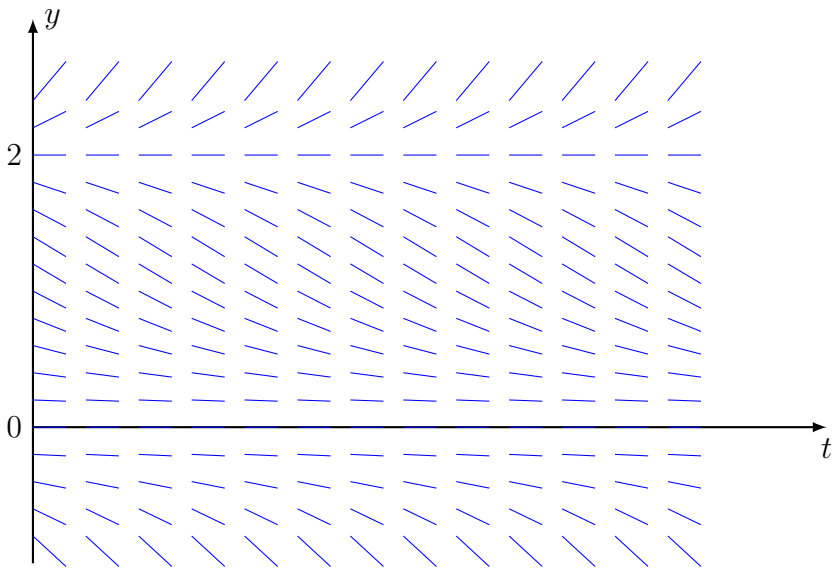
(C) $y' = y^2\,(2-y)$

(D) $y' = y^2\,(y-2)$

(E) $y' = y\,(y-2)^2$

(F) $y' = y^2\,(y-2)^2$

(G) None of the above.



Important: Choose only **one** option and write it in the table in page 2.

Answer: **(D)**, $y' = y^2\,(y-2)$

2. (5 points) **Question 2.**

What is the largest interval, I, in which the solution of

$$\cos(t)\, y'' + \sin(t)\, y' - \frac{5}{t}\, y = \frac{t^2}{(t-4)} \qquad y(2) = 2, \qquad y'(2) = 0.$$

is guaranteed to exist be the Existence and Uniqueness Theorem?

(A) $I = (-\infty, \infty)$

(B) $I = (0, \infty)$

(C) $I = (-\infty, 0) \cup \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, 4\right) \cup (0, \infty)$

(D) $I = \left(\frac{\pi}{2}, 4\right)$

(E) $I = \left(0, \frac{\pi}{2}\right)$

(F) $I = (0, 4)$

(G) $I = \left(1, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, 4\right) \cup (0, \infty)$

(H) None of the above.

Important: Choose only **one** option and write it in the table in page 2.

(D), $I = \left(\frac{\pi}{2}, 4\right)$

3. (5 points) **Question 3.**

Find the set of **stable** equilibrium solution(s) of

$$y' = y\,(y^2 - 4).$$

(A) $y(t)$ is any of -2, 0, 2

(B) $y(t)$ is any of 0, 2

(C) $y(t)$ is any of -2, 0

(D) $y(t)$ is any of -2, 2

(E) $y(t)$ is 2

(F) $y(t)$ is -2

(G) $y(t)$ is 0

(H) None of the above.

Important: Choose only **one** option and write it in the table in page 2.

(G), $y(t)$ is 0.

4. (5 points) **Question 4.**

Find the general solution of

$$y'' + 3y' + 2y = 4t.$$

(A) $y(t) = e^{-t} + e^{-2t} + 4t$

(B) $y(t) = c_1\, e^{t} + c_2\, e^{2t} + 4t$

(C) $y(t) = c_1\, e^{t} + c_2\, e^{2t} + 2t - 3$

(D) $y(t) = c_1\, e^{-t} + c_2\, e^{-2t} + 2t - 3$

(E) $y(t) = c_1\, e^{-t} + c_2\, e^{-2t} + 4t$

(F) None of the above.

Important: Choose only **one** option and write it in the table in page 2.

(D), $y(t) = c_1\, e^{-t} + c_2\, e^{-2t} + 2t - 3$.

5. (5 points) **Question 5.**

Find the Laplace transform of the function

$$f(t) = \begin{cases} 0, & t < 4 \\ t-4, & 4 < t \leqslant 8 \\ 0, & t \geqslant 8. \end{cases}$$

(A) $\mathcal{L}[f(t)] = e^{4s}\dfrac{1}{s^2} - e^{8s}\dfrac{1}{s^2} - 4\,e^{8s}\dfrac{1}{s}$

(B) $\mathcal{L}[f(t)] = e^{-4s}\dfrac{1}{s} - e^{-8s}\dfrac{1}{s}$

(C) $\mathcal{L}[f(t)] = e^{-4s}\dfrac{1}{s^2} - e^{-8s}\dfrac{1}{s^2}$

(D) $\mathcal{L}[f(t)] = e^{-4s}\dfrac{1}{s^2} - e^{-8s}\dfrac{1}{s^2} - 4\,e^{-8s}\dfrac{1}{s}$

(E) $\mathcal{L}[f(t)] = e^{4s}\dfrac{1}{s} - e^{8s}\dfrac{1}{s}$

(F) $\mathcal{L}[f(t)] = e^{4s}\dfrac{1}{s^2} - e^{8s}\dfrac{1}{s^2}$

(G) None of the above.

Important: Choose only **one** option and write it in the table in page 2.

(D), $\mathcal{L}[f(t)] = e^{-4s}\dfrac{1}{s^2} - e^{-8s}\dfrac{1}{s^2} - 4\,e^{-8s}\dfrac{1}{s}$.

6. (5 points) **Question 6.**

Find the function $f(t)$ such that

$$\mathcal{L}[f(t)] = \frac{e^{-5s}}{s^2 - 4s + 13}.$$

(A) $f(t) = \frac{1}{3}\, u(t-2)\, \sin(3(t-2))$

(B) $f(t) = u(t-2)e^{5(t-2)}\, \sin(3(t-2))$

(C) $f(t) = \frac{1}{3}\, u(t-2)e^{5(t-2)}\, \sin(3(t-2))$

(D) $f(t) = u(t-5)e^{2(t-5)}\, \sin(3(t-5))$

(E) $f(t) = \frac{1}{3}\, u(t-5)\, e^{2(t-5)}\, \sin(3(t-5))$

(F) $f(t) = \frac{1}{3}\, u(t-5)\, \sin(3(t-5))$

(G) None of the above.

Important: Choose only **one** option and write it in the table in page 2.

(E), $f(t) = \frac{1}{3}\, u(t-5)\, e^{2(t-5)}\, \sin(3(t-5))$.

7. (5 points) **Question 7.**

Find the general solution of

$$t\,y' - 2y = t^3\,e^{-t}.$$

(A) $y(t) = -e^{-t} + c$

(B) $y(t) = -t^2\,e^{-t}$

(C) $y(t) = -c\,e^{-t} + t^2$

(D) $y(t) = t\,e^{-t} + t$

(E) $y(t) = -t^2\,e^{-t} + c\,t^2$

(F) $y(t) = t^2\,e^{-t} + c\,t^2$

(G) $y(t) = -t\,e^{-t} + c\,t$

(H) None of the above.

Important: Choose only **one** option and write it in the table in page 2.

(E), $y(t) = -t^2\,e^{-t} + c\,t^2$.

8. (5 points) **Question 8.**

Suppose that $y_1(t)$ is solution of

$$y'' + a_1\, y' + a_0\, y = 0, \tag{1}$$

and $y_2(t)$ is solution of

$$y'' + a_1\, y' + a_0\, y = 4\, e^{3t}. \tag{2}$$

Which of the following statements are true?

(A) $y_1(t) - y_2(t)$ solves equation (1)

(B) $y_1(t) + y_2(t)$ solves equation (2)

(C) $-y_1(t) + y_2(t)$ solves equation (1)

(D) $-y_1(t) - y_2(t)$ solves equation (2)

(E) $3\, y_1(t)$ solves equation (1)

(F) $5\, y_1(t)$ solves equation (2)

(G) $7\, y_2(t)$ solves equation (2)

(H) $2\, y_2(t)$ solves equation (1)

(I) All of the above.

(J) None of the above.

Important: Choose **all** that apply and write it(them) in the table in page 2.

Answer:

- Full credit: **(B)** and **(E)**.
- No credit: Any other answer

9. (5 points) **Question 9.**

Which of the following equations are **linear** differential equations?

(A) $t\,y' + t - y^2 = \sin(t)$

(B) $t\,y' + y = \sin(y)$

(C) $y' = \dfrac{\tan(t)}{y\,e^t}$

(D) $t^3\,y' + y - t^2 = \cos(2t)$

(E) $y\,y'' + 3\,y' + 2\,y = \cos(t)$

(F) $y'' = y^3$

(G) $t\,y'' + \dfrac{y}{\sin(t)} = e^t$

(H) None of the above.

Important: Choose **all** that apply and write it(them) in the table in page 2.

Answer:

- Full credit: **(D)** and **(G)**.
- No credit: Any other answer

10. (20 points) **Question 10.**

Consider the following differential equation.

$$y' = 5(y-1)(y-3)(y-5).$$

(a) *(4 points)* Find the equilibrium solution(s).

(b) *(6 points)* Draw a phase diagram of the differential equation, clearly indicating the intervals where $y(t)$ is increasing and where it is decreasing. Label your axes.

(c) *(4 points)* Determine the stability of the equilibria.

(d) *(6 points)* Make rough sketches of representative solution curves with various initial conditions, one for each interval of increase/decrease. Label your axes.

(a) $y = 1$, $y = 3$, $y = 5$.

(b) Solution is increasing for $(1, 3) \cup (5, \infty)$, while solution is decreasing for $(-\infty, 1) \cup (3, 5)$.

(c) $y = 1$ is unstable, $y = 3$ is stable, and $y = 5$ is unstable.

(d) Draw a qualitative graph of a few solutions.

11. (20 points) **Question 11.** Use the Undetermined Coefficients Method to find the general solution of

$$y'' + y' + 3y = 3t^2.$$

Show all your work. Answers which are not substantiated will not receive full credit.

$$y(t) = c_1\, e^{-t/2} \cos\left(\frac{\sqrt{11}}{2}t\right) + c_2\, e^{-t/2} \sin\left(\frac{\sqrt{11}}{2}t\right) + t^2 - \frac{2}{3}\,t - \frac{4}{9}, \qquad c_1,\ c_2 \in \mathbb{R}.$$

12. (20 points) **Question 12.** Use the Laplace transform to find the solution to the initial value problem

$$y'' + 16y = 7u(t-2), \quad y(0) = 0, \quad y'(0) = 0.$$

Show all your work. Answers which are not substantiated will not receive full credit.

$$y(t) = \frac{7}{16}\, u(t-2)\left(1 - \cos(4(t-2))\right).$$

Congratulations you are now done with the exam!

Go back and check: - You copied your answers to MC questions in the MC table.

- Your solutions to problems 10, 11, and 12 are accurate and clear.
- Your answers to problems 10, 11, and 12 are BOXED.

When you are completely happy with your work please bring your exam to the front to be handed in.

Please have your MSU student ID ready so that is can be checked.

Students Do Not Write Below This Line.

Question	Points	Score
MC	45	
10	20	
11	20	
12	20	
Total	105	
Maximum	100	

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}, \quad n \neq -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

$$\int e^{ax}\,dx = \frac{e^{ax}}{a}, \qquad \int a^x\,dx = \frac{a^x}{\ln a}$$

$$\int \ln(ax)\,dx = x\,(\ln(ax) - 1)$$

$$\int x^n\,\ln(ax)\,dx = \frac{x^{(n+1)}}{(n+1)^2}\left[(n+1)\ln(ax) - 1\right]$$

$$\int x\,e^{ax}\,dx = \frac{e^{ax}}{a^2}\,(ax - 1)$$

$$\int x^2\,e^{ax}\,dx = \frac{e^{ax}}{a^3}\,(a^2x^2 - 2ax + 2)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\,\cos(ax)$$

$$\int \cos(ax)\,dx = \frac{1}{a}\,\sin(ax)$$

$$\int x\sin(ax)\,dx = -\frac{x}{a}\,\cos(ax) + \frac{1}{a^2}\,\sin(ax)$$

$$\int x\cos(ax)\,dx = \frac{x}{a}\,\sin(ax) + \frac{1}{a^2}\,\cos(ax)$$

$$\int e^{ax}\,\sin(bx)\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\,\sin(bx) - b\,\cos(bx)\right]$$

$$\int e^{ax}\,\cos(bx)\,dx = \frac{e^{ax}}{a^2+b^2}\left[b\,\sin(bx) + a\,\cos(bx)\right]$$

$$\int \tan(ax)\,dx = \frac{1}{a}\,\ln|\sec(ax)|$$

$$\int \sec^2(ax)\,dx = \frac{1}{a}\,\tan(ax)$$

$$\int \sec(ax)\,dx = \frac{1}{a}\,\ln|\sec(ax) + \tan(ax)|$$

$$\int \csc(ax)\,dx = -\frac{1}{a}\,\ln|\csc(ax) + \cot(ax)|$$

$$\int \sec(ax)\tan(ax)\,dx = \frac{1}{a}\,\sec(ax)$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\,\arctan\left(\frac{x}{a}\right)$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\left(\frac{x}{a}\right)$$

$$\int \frac{a}{x\sqrt{x^2-a^2}}\,dx = \operatorname{arcsec}\left(\frac{x}{a}\right)$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \cosh^{-1}\left(\frac{x}{a}\right) = \ln(x + \sqrt{x^2-a^2})$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \sinh^{-1}\left(\frac{x}{a}\right) = \ln(x + \sqrt{x^2+a^2})$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}, \qquad \cosh(x) = \frac{e^x + e^{-x}}{2}$$

$f(t)$	$F(s) = \mathcal{L}[f(t)]$	D_F
$f(t) = 1$	$F(s) = \frac{1}{s}$	$s > 0$
$f(t) = e^{at}$	$F(s) = \frac{1}{(s-a)}$	$s > a$
$f(t) = t^n$	$F(s) = \frac{n!}{s^{(n+1)}}$	$s > 0$
$f(t) = \sin(at)$	$F(s) = \frac{a}{s^2+a^2}$	$s > 0$
$f(t) = \cos(at)$	$F(s) = \frac{s}{s^2+a^2}$	$s > 0$
$f(t) = \sinh(at)$	$F(s) = \frac{a}{s^2-a^2}$	$s > \lvert a\rvert$
$f(t) = \cosh(at)$	$F(s) = \frac{s}{s^2-a^2}$	$s > \lvert a\rvert$
$f(t) = t^n e^{at}$	$F(s) = \frac{n!}{(s-a)^{(n+1)}}$	$s > \max\{a, 0\}$
$f(t) = e^{at}\sin(bt)$	$F(s) = \frac{b}{(s-a)^2+b^2}$	$s > \max\{a, 0\}$
$f(t) = e^{at}\cos(bt)$	$F(s) = \frac{(s-a)}{(s-a)^2+b^2}$	$s > \max\{a, 0\}$
$f(t) = e^{at}\sinh(bt)$	$F(s) = \frac{b}{(s-a)^2-b^2}$	$s > \max\{a, \lvert b\rvert\}$
$f(t) = e^{at}\cosh(bt)$	$F(s) = \frac{(s-a)}{(s-a)^2-b^2}$	$s > \max\{a, \lvert b\rvert\}$
$u(t-c)$	$\frac{e^{-cs}}{s}$	$s > 0,\ c \geqslant 0$
$\delta(t-c)$	e^{-cs}	$s \in \mathbb{R},\ c \geqslant 0$
$u(t-c)\,f(t-c)$	$e^{-cs}\,F(s)$	$c \geqslant 0$
$e^{ct}\,f(t)$	$F(s-c)$	$c \in \mathbb{R}$
$f'(t)$	$s\,F(s) - f(0)$	
$f''(t)$	$s^2\,F(s) - s\,f(0) - f'(0)$	
$(-t)^n\,f(t)$	$F^{(n)}(s)$	