Impulsive Forces

Impulsive forces instantly transfer large amounts of momentum

Team Members: 1. ____________________

2. ____________________

3. ____________________

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Objectives

To have a better understanding of impulsive forces.

Introduction

In physics, the momentum of a mechanical system with mass m and velocity $\boldsymbol{v}$ is the product $\boldsymbol{p} = m\boldsymbol{v}$. This product measures the inertia of the body, it contains the information of how fast it moves and how massive it is. It is not the same to stop a fly moving at 10 miles/hour than a truck moving at 10 miles/hour.

Impulsive forces change the momentum of objects in an instant. Some objects are strong enough to handle this, others not. We can (pretty much) instantly change the momentum of a golf ball. When we hit the ball with a golf club, the latter makes an impulsive force on the ball, and the ball can handle that. However, we cannot instantly change the momentum of a truck without destroying it. If a moving truck hits a concrete wall, the truck momentum changes pretty much instantly, but the truck is not the same after the impulsive force. The truck is too weak, it cannot handle the instant change.

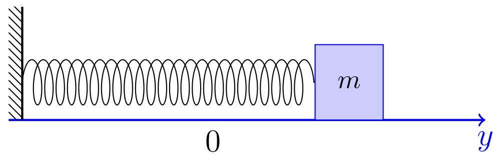
In this lab we will study the oscillations in a mass-spring system that is subject to an impulsive external force. And we will do the following.

- First, we prove that impulsive forces transfer large amounts of momentum in an instant. We do it in a simple way: We solve two different initial value problem and we show that they have exactly the same solution, except at one point.

- Then, we use an impulsive force for a specific task: to stop in an instant an oscillating mass-spring system. In order to do that, we need to choose the right place to act with the impulsive for, we need to choose the right force intensity, and the right force direction.

Instant Change in Momentum

A mass-spring system is a mass m attached to a spring with spring constant k that slides on a frictionless table—for example a table covered with a flat, smooth, ice surface. We denote by $y(t)$ the displacement of the mass as a function of time, where $y = 0$ represents the rest position of the mass.



This system is described by Newton's law of motion $ma = f$, where $a = y''$, and f is given by Hooke's law, $f(y) = -k\,y$. So the equation of motion for the oscillating spring without any external force is

$$m\,y'' + k\,y = 0 \quad \Rightarrow \quad y'' + \omega^2 y = 0, \qquad \omega^2 = \frac{k}{m}.$$

where ω is the **natural frequency** of the oscillator.

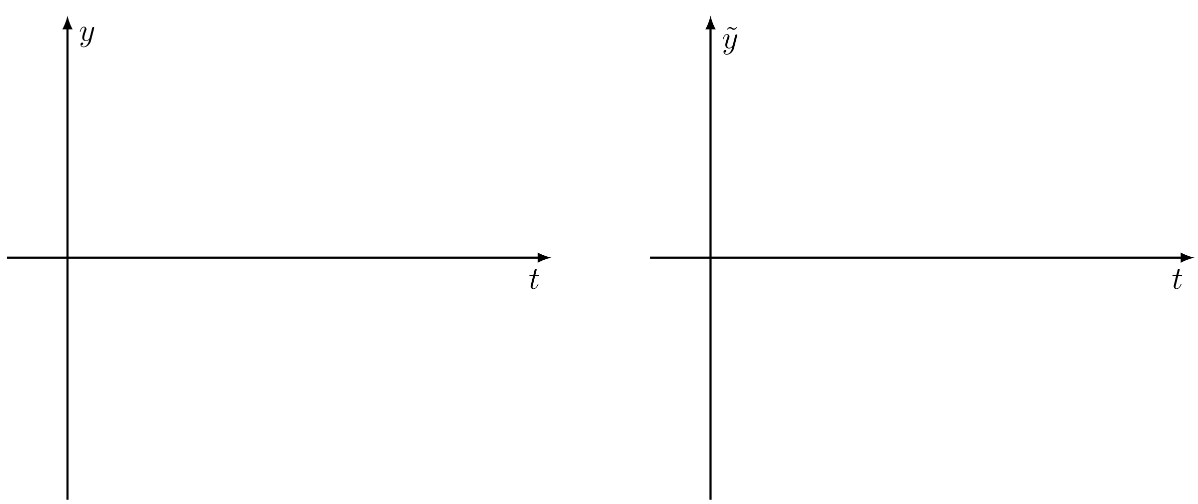
Question 1. *(1 point)* Use the Laplace transform to solve the initial value problem for y,

$$y'' + \omega^2 y = 0, \qquad y(0) = 0, \qquad y'(0) = 1.$$

Question 2. *(1 point)* Use the Laplace transform to solve the initial value problem for $\tilde{y}$,

$$\tilde{y}'' + \omega^2 \tilde{y} = \delta(t), \qquad \tilde{y}(0) = 0, \qquad \tilde{y}'(0) = 0.$$

Question 3. *(1 points)* Plot the graph of y' and $\tilde{y}'$, as function of time, and see if you can spot the difference.



Stopping an Oscillation Instantly

Consider a mass-spring system that moves according to the solution of the initial value problem

$$y'' + y = F_0\,\delta\Big(t - \frac{\pi}{2}\Delta t\Big), \qquad \Delta t > 0, \qquad y(0) = y_0, \qquad y'(0) = 0.$$

Question 4. *(3 points)* Find the solution of the initial above for arbitrary constants F_0 and Δt.

In the interactive graph below we plot the solution found question 4. The graph has three parameters:

- Δt, the (positive) time that measures when we apply the impulsive force;
- F_0, the intensity of the impulsive force, which is positive to the right and negative to the left;
- y_0, the initial amplitude of the mass-spring system, again positive to the right and negative to the left.

Use the interactive graph to see how the solution of question 4 changes when we change Δt, F_0, y_0.

Interactive Graph Link.

Use the interactive graph and the trigonometric identity

$$\sin(\theta + \phi) = \sin(\theta)\,\cos(\phi) + \sin(\phi)\,\cos(\theta).$$

to help you answer the following questions:

Question 5. *(2 points)* Given an arbitrary value for the initial amplitude $y_0 \neq 0$, find the value of F_0 and the smallest value of $\Delta t > 0$ such that the impulsive force stops the mass instantly.

Question 6. *(2 points)* Given an arbitrary value for the initial amplitude y_0, find the value of F_0 and the second smallest value of $\Delta t > 0$ such that the impulsive force stops the mass instantly.