Introduction to Modeling

Discrete and Continuous Models of Population Dynamics

Team Members: 1. ____________________

2. ____________________

3. ____________________

4. ____________________

Objectives

By the end of this lab students should be able to create mathematical descriptions of population systems.

Introduction

In this Lab we do the following:

- We start with a "life and death" experiment on virtually tossed pennies. We find a discrete equation for the number of pennies at each toss.
- We generalize the "life and death" experiment adding a fixed immigration quantity at every toss. We find a discrete equation for the number of pennies at each toss.
- Given a discrete equation for a system we show how to obtain a differential equation that describe the same system.
- Finally, given different physical systems we find their corresponding discrete equations and the differential equations that describe them.

Part 1: Tossing Pennies (5min)

We virtually toss 50 pennies using `www.random.org/coins`. If a penny lands tails up, we consider it dead and remove it from the population. Otherwise, if it lands heads up, it gets to live another round. After each round, we repeat the experiment with the surviving pennies. For example, if 30 pennies land heads up, then we consider that 30 of them have survived and at the next round we toss 30 pennies.

1. Without conducting the virtual experiment, predict how many rounds the experiment may last before all the pennies are removed from the population.

2. Do the experiment and write down your results in the Table 1 below.

Table 1: Modeling Pennies

Iteration	# of pennies at start of iteration
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

3. Find a discrete equation that describes the average behavior of the system. A discrete equation relates $P(n+1)$, the number of pennies heads up at the iteration $n+1$, with $P(n)$, the number of pennies heads up at the iteration n.

Part 2: Tossing Pennies with Immigration (5min)

Once again we start with 50 pennies and we virtually toss them using www.random.org/coins. Just as before, if a penny lands tails up, we consider it dead and remove it from the population. Otherwise, if it lands heads up, it gets to live another round. The new part in this experiment is that, before tossing the next round we add 10 new pennies, called the immigrant pennies. Only then we toss together the surviving pennies and the immigrant pennies. We repeat this procedure 10 times.

1. Without conducting the virtual experiment, predict how many rounds the experiment may last before all the pennies are removed from the population.

2. Do the experiment and write down your results in the Table 2 below.

Table 2: Modeling Pennies with Immigration

Iteration	# of pennies at start of iteration
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

3. Find a discrete equation that describes the average behavior of the system. A discrete equation relates $P(n+1)$, the number of pennies heads up at the iteration $n+1$, with $P(n)$, the number of pennies heads up at the iteration n.

Part 3: Continuum Models for Population Systems (20 min)

Our system now is the population of a particular species of fish in a lake. The initial population is 1000 individuals. The fish population after a time period of 1 month is given by the fish population of the previous month minus 20%. Also, 30 fish individuals immigrate into the lake every month.

1. Find the discrete equation satisfied by $P(n+1)$, the fish population after $(n+1)$ months, as a function of $P(n)$.

2. How is this related to differential equations? We show in class how this discrete equation is related to an ordinary differential equation—by finding the continuum limit of the discrete equation above.

3. From the continuum equation, can you determine the equilibrium solution in this case (i.e., the number of fish at which the population will level off)? Recall, an equilibrium solution is a constant, it does not depend on time.

4. Solve the differential equation above. Recall that $\ln\big(f(t)\big)' = \dfrac{f'(t)}{f(t)}$.

Part 4: Modeling Different Population Systems (15 min)

Write the discrete and continuum equations for the following systems.

1. Consider the population of salmon in Lake Michigan. If left unmanaged, the salmon population decreases by a 25% each year. To support the population, the Michigan DNR stocks the lake with 2,000 every year.

2. Consider the population of worms in a composting pile. Assume the number of worms doubles (increases by 100%) each week and that a farmer adds 10 worms to the pile each week.

3. Consider a population of rabbits in a forest. Assume the rabbits triple their numbers after each year. Assume that the Michigan DNR has issued hunting permits for 50 rabbits to be shot each year.