Section 5.3 : Matrix Algebra

Plan: * Determinant of 3x3 Matrices

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* Determinant of 3x3 Matrices

Remark: * The $\det(A)$, A 2×2, determines whether A is invertible or not.

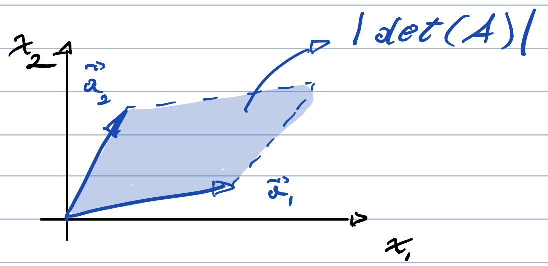
* One can find a similar number $\det(A)$ for $n\times n$ matrices, that determines whether an $n\times n$ matrix is invertible.

* Using the geometrical meaning of $\det(A)$.

Geometrical Meaning of 2x2 Det. :

Given $A = [\vec{a}_1, \vec{a}_2]$, 2×2,

$|\det(A)|$ = Area of parallelogram formed by $\vec{a}_1$, $\vec{a}_2$.



We use this geometric interpretation of det. of 2×2 matrices to find dets. of 3×3 matrices.

Generalization of Det. from 2x2 to 3x3 matrices:

Given $A = [\vec{a}_1, \vec{a}_2, \vec{a}_3]$, 3x3,

one can define $\det(A)$ so that

$$|\det(A)| = \text{Volume of parallelepiped formed by } \vec{a}_1, \vec{a}_2, \vec{a}_3 .$$

Remark: Then, one finds a recursive formula.

Def.:
$$\left[\ \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = + a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}\ \right]$$

Example: $\left[\ \text{Find } \det(A),\quad A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix}\ \right]$

Sol:

$$\begin{vmatrix} 1 & 3 & -1 \\ 2 & 1 & 1 \\ 3 & 2 & 1 \end{vmatrix} = 1 \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} - 3 \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} + (-1) \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}$$

$$= 1(1-2) - 3(2-3) - (1)(4-3)$$

$$= -1 + 3 - 1$$

$$= 1$$