

* Solving IVP with Generalized Sources

Example:
$$\left[\begin{array}{l} \text{Solve: } y'' + 25y = 2 \cos(t) \delta(t-\pi) \\ y(0) = 0, \quad y'(0) = 0 \end{array} \right]$$

Sol:

$$\mathcal{L}[y'' + 25y] = \mathcal{L}[2 \cos(t) \delta(t-\pi)]$$

$$\mathcal{L}[y''] + 25 \mathcal{L}[y] = 2 \cos(\pi) e^{-\pi s}$$

$$s^2 \mathcal{L}[y] + 25 \mathcal{L}[y] = -2 e^{-\pi s}$$

$$(s^2 + 25) \mathcal{L}[y] = -2 e^{-\pi s}$$

$$\boxed{\mathcal{L}[y] = e^{-\pi s} \frac{(-2)}{s^2 + 5^2}}$$

$$\mathcal{L}[y] = \frac{-2}{5} e^{-\pi s} \left(\frac{5}{s^2 + 5^2} \right)$$

$$= -\frac{2}{5} e^{-\pi s} \mathcal{L}[\sin(5t)]$$

$$= -\frac{2}{5} \mathcal{L}[u(t-\pi) \sin(5(t-\pi))]$$

$$\boxed{y(t) = -\frac{2}{5} u(t-\pi) \sin 5(t-\pi)}$$