The Bernoulli Equation

We transform a nonlinear equation into a linear equation

Objectives

Students should be able to identify and solve a Bernoulli equation.

Introduction

The **Bernoulli equation** with coefficients functions p, q, and index $n \in \mathbb{R}$ is given by

$$y' = p(t)\,y + q(t)\,y^n. \tag{1}$$

- For $n \neq 0, 1$ the equation is nonlinear.
- If $n = 2$ we get the *logistic equation*.
$$y' = ry\left(1 - \frac{y}{K}\right).$$
- The Bernoulli equation in this sections is not the Bernoulli equation from fluid dynamics.
- The Bernoulli equation is special because *it is a nonlinear equation that can be transformed into a linear equation.*

Theorem 1. The function y is a solution of the Bernoulli equation

$$y' = p(t)\, y + q(t)\, y^n, \qquad n \neq 1,$$

iff the function $v = 1/y^{(n-1)}$ is solution of the linear differential equation

$$v' = -(n-1)p(t)\, v - (n-1)q(t).$$

Question 1. *(2 points)* Prove Theorem 1.

□

Question 2. *(4 points)* Find every nonzero solution of the differential equation

$$y' = y + 3\,y^4.$$

Question 3. *(4 points)* Find every solution of the equation $t\,y' = 3y + \frac{9}{2}\,t^5\,y^{1/3}$.