Name: ______________________ **PID:** ______________

Section: ____ **Recitation Instructor**: ______________________

READ THE FOLLOWING INSTRUCTIONS.

- **Do not open your exam until told to do so.**
- Without fully opening the exam, check that you have pages 1 through 21.
- Fill in your name, etc, on this first page and **sign** at the bottom.
- Write your answers for the **Multiple Choice** (MC) problems **in the table on page 2**.
- In the **Show Your Work** (SYW) problems, **problems 16 and 17**, you must show all your work. Write your answers clearly. Include enough steps for you to find possible mistakes when you revise your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- If you need scratch paper, use the back of the previous page.
- First do the problems you know how to do. Do not spend too much time on any particular problem. Return to the difficult problems later.
- If you have any questions please raise your hand and a proctor will come to you.
- You will be given exactly **120 minutes** for this exam.

ACADEMIC DISHONESTY.

- **No calculators, no phones, or any other electronic devices can be used on this exam**.
- Clear your desk of everything excepts pens, pencils and erasers.
- There is **no talking** allowed during the exam. Please **do not look at other students papers**.
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe. All cases of academic dishonesty will be reported immediately to the Dean of Undergraduate Studies and added to the student's academic record.

I have read and understand the above instructions: ______________________

SIGNATURE

Answers to Multiple Choice Questions.

Students must **write their answers** to the Multiple Choice questions in the table below.

Instructors **will not look** at the pages with the Multiple Choice questions.

Instructors **will look only at this table** to grade your answers to the Multiple Choice questions.

Students Answers to Multiple Choice Questions

Question	Points	**Student Answers Here**	Score (for TA only)
1	5	**(E)**	
2	5	**(F)**	
3	5	**(F), (H), (I)**	
4	5	**(E)**	
5	5	**(B)**	
6	5	**(D)**	
7	5	**(A)**	
8	5	**(E)**	
9	5	**(C)**	
10	5	**(F)**	
11	5	**(H)**	
12	5	**(A)**	
13	5	**(H)**	
14	5	**(B)**	
15	5	**(C)**	
Total	75	Do not write in this box	

(1) *(5 points)* A bacteria culture grows at a rate proportional to the amount present. There are 20 strands of bacteria at $t = 0$ and 60 strands after 4 hours. Find $N(t)$, the **amount of bacteria** strands as function of time in hours.

(A) $N(t) = 60\, e^{4t}$.

(B) $N(t) = 60\, e^{-4t}$.

(C) $N(t) = 20\, e^{t/4}$.

(D) $N(t) = 20\, e^{-t/4}$.

(E) $N(t) = 20\, e^{t\ln(3)/4}$.

(F) $N(t) = 20\, e^{-t\ln(3)/4}$.

(G) $N(t) = 60\, e^{t/\ln(4)}$.

(H) $N(t) = 60\, e^{-t/\ln(4)}$.

(I) $N(t) = 20\, e^{t\ln(4)}$.

(J) $N(t) = 20\, e^{-t\ln(4)}$.

(K) There is no way to compute $N(t)$ from the data provided.

(L) None of the above.

Important: Choose only **one** option and write it in the table on page 2.

Answer: **(E)**

(2) *(5 points)* Identify the **differential equation** that produces the slope field below.

(A) $y' = y(y+2)$

(B) $y' = y(y-2)$

(C) $y' = y(y+2)^2$

(D) $y' = y(y-2)^2$

(E) $y' = y(y+2)^2$

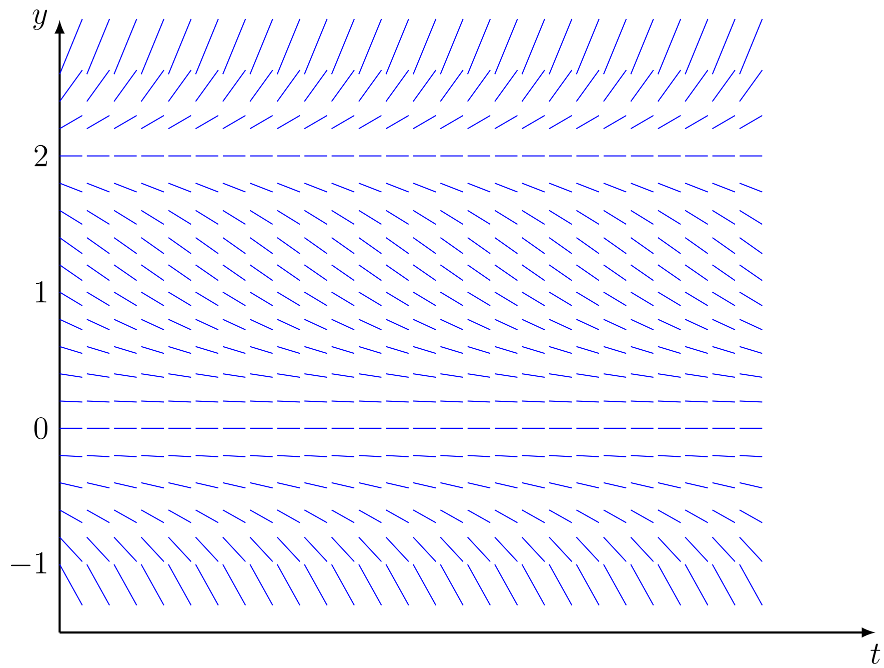
(F) $y' = y^2(y-2)$

(G) $y' = y^2(y+2)$

(H) $y' = y^2(y-2)^2$

(I) $y' = y^2(y+2)^2$

(J) None of the above.



Important: Choose only **one** option and write it in the table on page 2.

Answer: **(F)**

(3) *(5 points)* Consider the initial value problem

$$y' = -y\,(y-1)\,(y-2), \qquad y(0) = y_0.$$

For which value(s) of y_0 the corresponding solution is **not constant** and **satisfies** $\lim_{t\to\infty} y(t) = 2$?

(A) $y_0 = -1$

(B) $y_0 = -0.5$

(C) $y_0 = 0$

(D) $y_0 = 0.5$

(E) $y_0 = 1$

(F) $y_0 = 1.5$

(G) $y_0 = 2$

(H) $y_0 = 2.5$

(I) $y_0 = 3$

(J) None of the above.

Important: Choose **all options that apply** and write it/them in the table on page 2.

Answer: **(F)**, **(H)**, **(I)**

(4) *(5 points)* Find $y(2)$, where $y(t)$ is the solution of the initial value problem

$$2ty\, y' = 3y^2 + t^2, \qquad y(1) = 1, \quad t > 0.$$

(A) $y(2) = 2$

(B) $y(2) = -2$

(C) $y(2) = 2\sqrt{2}$

(D) $y(2) = -2\sqrt{2}$

(E) $y(2) = 2\sqrt{3}$

(F) $y(2) = -2\sqrt{3}$

(G) $y(2) = 2\sqrt{5}$

(H) $y(2) = -2\sqrt{5}$

(I) $y(2) = 1$

(J) None of the above.

Important: Choose only **one** option and write it in the table on page 2.

Answer: **(E)**

(5) *(5 points)* Find $y(2)$, where $y(t)$ is the solution of the initial value problem

$$t\,y' + y - t^2 = 0, \qquad y(1) = 1, \quad t > 0.$$

(A) $y(2) = 2$

(B) $y(2) = \dfrac{5}{3}$

(C) $y(2) = -\dfrac{5}{3}$

(D) $y(2) = \dfrac{5}{4}$

(E) $y(2) = -\dfrac{5}{4}$

(F) $y(2) = 10\,e^{-2} - 9$

(G) $y(2) = 10\,e^{2} - 9$

(H) $y(2) = -5\,e^{-2} + 6$

(I) $y(2) = -5\,e^{2} + 6$

(J) None of the above.

Important: Choose only **one** option and write it in the table on page 2.

Answer: **(B)**

(6) *(5 points)* Find the largest interval D in which the solution $y(t)$ of the initial value problem below is **guaranteed** to exist by the Existence and Uniqueness Theorem,

$$\cos(t)\, y'' + t^2\, y' - \frac{5}{t}\, y = \frac{e^t}{t-3}, \qquad y(1) = 2, \quad y'(1) = 0.$$

(A) $D = (-\infty, \infty)$

(B) $D = (0, \infty)$

(C) $D = (-\infty, 0)$

(D) $D = \left(0, \frac{\pi}{2}\right)$

(E) $D = \left(-\frac{\pi}{2}, 0\right)$

(F) $D = \left(\frac{\pi}{2}, 3\right)$

(G) $D = (0, \pi)$

(H) $D = (2, \pi)$

(I) $D = (3, \pi)$

(J) None of the above.

Important: Choose only **one** option and write it in the table on page 2.

Answer: **(D)**

(7) *(5 points)* If $W \subset \mathbb{R}^3$ is given by

$$W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ so that } -2x + 4y - 6z = 2 \right\},$$

then choose one of the options below.

(A) A basis of W is: $\left\{ \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} \right\}$.

(B) A basis of W is: $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right\}$.

(C) A basis of W is: $\left\{ \begin{bmatrix} -2 \\ 4 \\ -6 \end{bmatrix} \right\}$.

(D) A basis of W is: $\left\{ \begin{bmatrix} -2 \\ 4 \\ -6 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \right\}$.

(E) A basis of W is: $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$.

(F) A basis of W is: $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

(G) A basis of W is: $\left\{ \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$.

(H) There is no basis because W is not a subspace.

(I) None of the above.

Important: Choose only **one** option and write it in the table on page 2.

Answer: **(H)**

(8) *(5 points)* An object of mass 6 grams is hanging at the bottom of a spring with spring constant 4 grams per second square. Assume there is **no dissipation** in the system. Denote by y the vertical coordinate of the object, positive downwards, and set $y = 0$ at the spring-mass rest position. Find the **maximum value of the velocity**, $v_{\max}$, achieved during the motion of the object with initial position $y(0) = -1$ centimeters and initial velocity $y'(0) = 2$ centimeters per second.

(A) $v_{\max} = \sqrt{\dfrac{14}{3}}$

(B) $v_{\max} = \sqrt{\dfrac{11}{2}}$

(C) $v_{\max} = 2$

(D) $v_{\max} = 1$

(E) $v_{\max} = 6$

(F) $v_{\max} = 4$

(G) $v_{\max} = \sqrt{2}$

(H) $v_{\max} = \sqrt{\dfrac{2}{3}}$

(I) There is no way to compute $v_{\max}$ from the data provided.

(J) None of the above.

Important: Choose only **one** option and write it in the table on page 2.

Answer: **(A)**

(9) *(5 points)* Find the general solution, $y(t)$, of the differential equation

$$y'' + 4\,y' + 5\,y = 0.$$

(A) $y(t) = c_1\,e^{-3t} + c_2\,e^{-t}$.

(B) $y(t) = c_1\,e^{3t} + c_2\,e^{t}$.

(C) $y(t) = c_1\,e^{-3t} + c_2\,t\,e^{-3t}$.

(D) $y(t) = c_1\,e^{-t} + c_2\,t\,e^{-t}$.

(E) $y(t) = c_1\,e^{-2t}\cos(t) + c_2\,e^{-2t}\sin(t)$.

(F) $y(t) = c_1\,e^{2t}\cos(t) + c_2\,e^{2t}\sin(t)$.

(G) $y(t) = c_1\,e^{t}\cos(-2t) + c_2\,e^{t}\sin(-2t)$.

(H) $y(t) = c_1\,e^{5t}\cos(4t) + c_2\,e^{5t}\sin(4t)$.

(I) $y(t) = c_1\,e^{4t}\cos(5t) + c_2\,e^{4t}\sin(5t)$.

(J) There is no way to compute $y(t)$ from the data provided.

(K) None of the above.

Important: Choose only **one** option and write it in the table on page 2.

Answer: **(E)**

(10) *(5 points)* Find a particular solution, $y(t)$, of the differential equation

$$y'' + 2\,y' + y = e^{-t}.$$

(A) $y(t) = \frac{1}{2}\,e^{-t}$.

(B) $y(t) = \frac{1}{2}\,t\,e^{-t}$.

(C) $y(t) = \frac{1}{2}\,t^2\,e^{-t}$.

(D) $y(t) = \frac{1}{2}\,t^3\,e^{-t}$.

(E) $y(t) = c_1\,e^{-t} + c_2\,t\,e^{-t}$.

(F) $y(t) = e^{-t} + 2\,t\,e^{-t}$.

(G) $y(t) = 2\,e^{-t} + t\,e^{-t}$.

(H) $y(t) = 2\,t^2\,e^{-t}$.

(I) $y(t) = 2\,t^3\,e^{-t}$.

(J) There is no way to compute $y(t)$ from the data provided.

(K) None of the above.

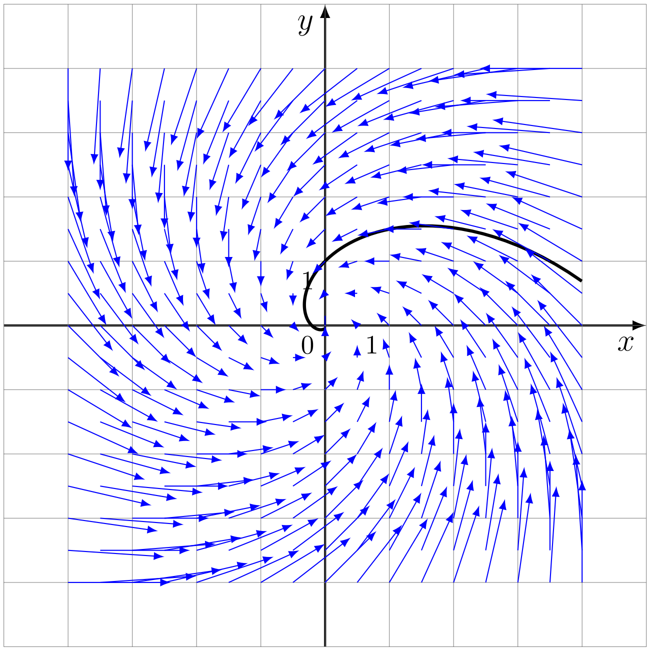
Important: Choose only **one** option and write it in the table on page 2.

Answer: **(C)**

(11) *(5 points)* Find the eigenpairs of matrix A and the vector $\boldsymbol{x}_0$ such that the **initial value problem**

$$\boldsymbol{x}'(t) = A\boldsymbol{x}(t), \qquad \boldsymbol{x}(0) = \boldsymbol{x}_0,$$

has the solution curve displayed in the phase portrait.



(A) $\lambda_\pm = -2 \pm 3i, \quad \boldsymbol{v}_\pm = \begin{bmatrix}1\\0\end{bmatrix} \pm \begin{bmatrix}0\\-1\end{bmatrix} i, \quad \boldsymbol{x}_0 = \begin{bmatrix}1\\0\end{bmatrix}.$

(B) $\lambda_\pm = -3 \pm 2i, \quad \boldsymbol{v}_\pm = \begin{bmatrix}0\\1\end{bmatrix} \pm \begin{bmatrix}1\\0\end{bmatrix} i, \quad \boldsymbol{x}_0 = \begin{bmatrix}0\\1\end{bmatrix}.$

(C) $\lambda_\pm = 2 \pm 3i, \quad \boldsymbol{v}_\pm = \begin{bmatrix}1\\0\end{bmatrix} \pm \begin{bmatrix}0\\-1\end{bmatrix} i, \quad \boldsymbol{x}_0 = \begin{bmatrix}0\\1\end{bmatrix}.$

(D) $\lambda_\pm = -3 \pm 2i, \quad \boldsymbol{v}_\pm = \begin{bmatrix}1\\0\end{bmatrix} \pm \begin{bmatrix}0\\1\end{bmatrix} i, \quad \boldsymbol{x}_0 = \begin{bmatrix}0\\1\end{bmatrix}.$

(E) $\lambda_\pm = 2 \pm 3i, \quad \boldsymbol{v}_\pm = \begin{bmatrix}1\\0\end{bmatrix} \pm \begin{bmatrix}0\\1\end{bmatrix} i, \quad \boldsymbol{x}_0 = \begin{bmatrix}1\\1\end{bmatrix}.$

(F) None of the above.

Important: Choose only **one** option and write it in the table on page 2.

Answer: **(B)**

(12) *(5 points)* Find the function $f(t)$ such that its Laplace transform is

$$\mathcal{L}[f(t)] = \frac{(s-11)\,e^{-2s}}{(s-3)^2+16}.$$

(A) $f(t) = u(t-2)\big(\cos(4t) - 8\sin(4t)\big)$.

(B) $f(t) = u(t-2)\big(\cos(3t) - 11\sin(3t)\big)$.

(C) $f(t) = u(t-2)\,e^{3t}\big(\cos(4t) - 2\sin(4t)\big)$.

(D) $f(t) = e^{3(t-2)}\big(\cos(4(t-2)) - 8\sin(4(t-2))\big)$.

(E) $f(t) = u(t-3)\,e^{2(t-3)}\big(\cos(4(t-3)) - 2\sin(4(t-3))\big)$.

(F) $f(t) = u(t-2)\,e^{3(t-2)}\big(\cos(4(t-2)) - 2\sin(4(t-2))\big)$.

(G) $f(t) = u(t-2)\,\big(\cos(4(t-3)) - 8\sin(4(t-3))\big)$.

(H) $f(t) = u(t-2)\,\big(\cos(4(t-3)) - 2\sin(4(t-3))\big)$.

(I) None of the above.

Important: Choose only **one** option and write it in the table on page 2.

Answer: **(F)**

(13) *(5 points)* Find $y(t)$ the solution of the initial value problem

$$y'' - 3\,y' + 2\,y = \cos\Big(\frac{\pi}{2}\,t\Big)\,\delta(t-2), \qquad y(0) = 0, \quad y'(0) = 0.$$

(A) $y(t) = e^{t} - e^{2t}$.

(B) $y(t) = -e^{t} + e^{2t}$.

(C) $y(t) = e^{t} - e^{2t}$.

(D) $y(t) = e^{t} - e^{2t}$.

(E) $y(t) = u(t-2)\big(-e^{t} + e^{2t}\big)$.

(F) $y(t) = u(t-2)\big(e^{t} - e^{2t}\big)$.

(G) $y(t) = u(t-2)\big(-e^{(t-2)} + e^{2(t-2)}\big)$.

(H) $y(t) = u(t-2)\big(e^{(t-2)} - e^{2(t-2)}\big)$.

(I) $y(t) = u(t+2)\big(-e^{(t+2)} + e^{2(t+2)}\big)$.

(J) $y(t) = u(t+2)\big(e^{(t+2)} - e^{2(t+2)}\big)$.

(K) None of the above.

Important: Choose only **one** option and write it in the table on page 2.

Answer: **(H)**

(14) *(5 points)* Classify the critical point $(1, 1)$ of the nonlinear system

$$\boldsymbol{x}' = \boldsymbol{f}(\boldsymbol{x}), \quad \text{where} \quad \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} x_1(2 - x_1) - x_1 x_2 \\ x_2(3 - x_2) - 2x_1 x_2 \end{bmatrix}.$$

(A) Sink Node.

(B) Source Node.

(C) Saddle Node.

(D) Sink Spiral.

(E) Source Spiral.

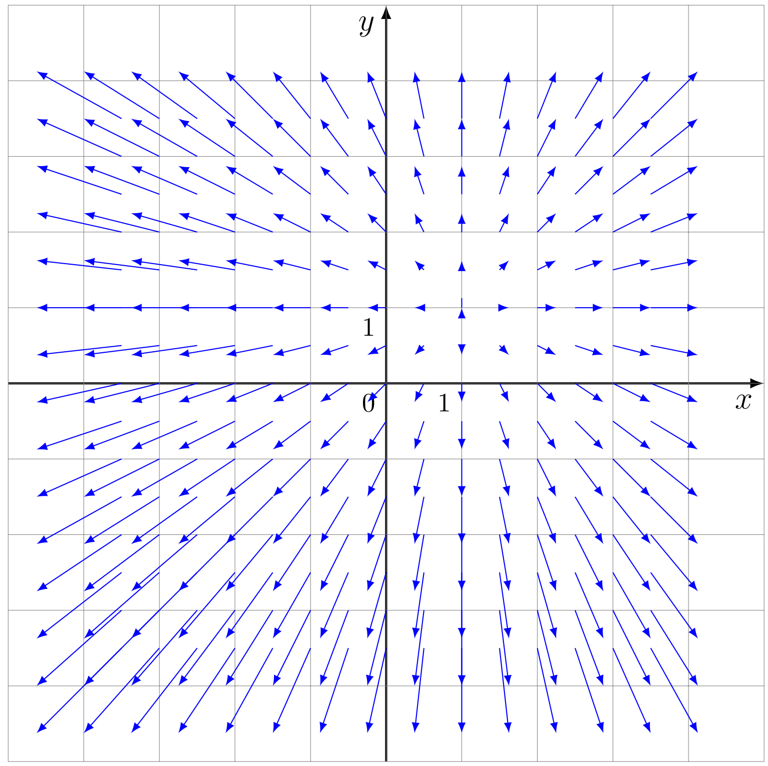
(F) Center.

(G) None of the above.

Important: Choose only **one** option and write it in the table on page 2.

Answer: **(C)**

(15) *(5 points)* Find the system of differential equations having the vector field shown in the picture.



(A) $x' = x - 1$
$y' = y - 1$

(B) $x' = -y$
$y' = x$

(C) $x' = -1 + y$
$y' = 1 - x$

(D) $x' = -y$
$y' = -x$

(E) $x' = x + 1$
$y' = y + 1$

(F) $x' = x - 1$
$y' = y + 1$

(G) None of the above.

Important: Choose only **one** option and write it in the table on page 2.

Answer: **(A)**

(16) *(15 points)* **Show Your Work Problem**

Assume that all solutions of a heat equation with appropriate boundary conditions are

$$u(t,x) = \sum_{n=1}^{\infty} c_n \, e^{-n^2\pi^2 t} \, \sin(n\pi x),$$

where c_n are arbitrary constants. From the solutions above find the one that satisfies the initial condition

$$u(0,t) = \begin{cases} 0, & x \in \left[0, \frac{1}{4}\right) \\ 1, & x \in \left[\frac{1}{4}, \frac{3}{4}\right) \\ 0, & x \in \left[\frac{3}{4}, 1\right]. \end{cases}$$

Answer: $\boxed{u(t,x) = \sum_{n=1}^{\infty} \frac{2}{n\pi}\Big(\cos\Big(\frac{n\pi}{4}\Big) - \cos\Big(\frac{3n\pi}{4}\Big)\Big) \, e^{-n^2\pi^2 \, t} \, \sin(n\pi x)}$.

(17) *(15 points)* **Show Your Work Problem**

Find all solutions $u(t,x)$ of the heat equation

$$\partial_t u = k\,\partial_x^2 u, \qquad t \in (0,\infty), \quad x \in (0,\ell),$$

where $k > 0$ and $\ell > 0$ are given constants, satisfying the boundary conditions

$$u(t,0) = 0, \quad \partial_x u(t,\ell) = 0, \qquad t \in [0,\infty).$$

Answer: $u(t,x) = \sum_{n=1}^{\infty} c_n\, e^{-k(\frac{(2n-1)\pi}{2\ell})^2\, t}\, \sin\Big(\frac{(2n-1)\pi}{2\ell}x\Big)$, where c_n are arbitrary constants.

Congratulations you are now done with the exam!

Go back and check: - You copied your answers to MC questions in the MC table.

- Your solutions to SYW problems 16 and 17 are accurate and clear.

- Your answers to SYW problems 16 and 17 are BOXED.

When you are completely happy with your work please bring your exam to the front to be handed in.

Please have your MSU student ID ready so that is can be checked.

Students Do Not Write Below This Line.

Question	Points	Score
MC	75	
16	15	
17	15	
Total	105	
Maximum	100	

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}, \quad n \neq -1; \quad \int \frac{1}{x}\,dx = \ln|x|$$
$$\int e^{ax}\,dx = \frac{e^{ax}}{a}, \qquad \int a^x\,dx = \frac{a^x}{\ln a}$$
$$\int \ln(ax)\,dx = x\,(\ln(ax) - 1)$$
$$\int x^n\,\ln(ax)\,dx = \frac{x^{(n+1)}}{(n+1)^2}\left[(n+1)\ln(ax) - 1\right]$$
$$\int x\,e^{ax}\,dx = \frac{e^{ax}}{a^2}\,(ax - 1)$$
$$\int x^2\,e^{ax}\,dx = \frac{e^{ax}}{a^3}\,(a^2x^2 - 2ax + 2)$$
$$\int \sin(ax)\,dx = -\frac{1}{a}\,\cos(ax)$$
$$\int \cos(ax)\,dx = \frac{1}{a}\,\sin(ax)$$
$$\int x\sin(ax)\,dx = -\frac{x}{a}\,\cos(ax) + \frac{1}{a^2}\,\sin(ax)$$
$$\int x\cos(ax)\,dx = \frac{x}{a}\,\sin(ax) + \frac{1}{a^2}\,\cos(ax)$$
$$\int e^{ax}\,\sin(bx)\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\,\sin(bx) - b\,\cos(bx)\right]$$
$$\int e^{ax}\,\cos(bx)\,dx = \frac{e^{ax}}{a^2+b^2}\left[b\,\sin(bx) + a\,\cos(bx)\right]$$
$$\int \tan(ax)\,dx = \frac{1}{a}\,\ln|\sec(ax)|$$
$$\int \sec^2(ax)\,dx = \frac{1}{a}\,\tan(ax)$$
$$\int \sec(ax)\,dx = \frac{1}{a}\,\ln|\sec(ax) + \tan(ax)|$$
$$\int \csc(ax)\,dx = -\frac{1}{a}\,\ln|\csc(ax) + \cot(ax)|$$
$$\int \sec(ax)\tan(ax)\,dx = \frac{1}{a}\,\sec(ax)$$
$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\,\arctan\Big(\frac{x}{a}\Big)$$
$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\Big(\frac{x}{a}\Big)$$
$$\int \frac{a}{x\sqrt{x^2-a^2}}\,dx = \mathrm{arcsec}\Big(\frac{x}{a}\Big)$$
$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \cosh^{-1}\Big(\frac{x}{a}\Big) = \ln(x + \sqrt{x^2-a^2})$$
$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \sinh^{-1}\Big(\frac{x}{a}\Big) = \ln(x + \sqrt{x^2+a^2})$$
$$\sinh(x) = \frac{e^x - e^{-x}}{2}, \qquad \cosh(x) = \frac{e^x + e^{-x}}{2}$$

$f(t)$	$F(s) = \mathcal{L}[f(t)]$	D_F
$f(t) = 1$	$F(s) = \frac{1}{s}$	$s > 0$
$f(t) = e^{at}$	$F(s) = \frac{1}{(s-a)}$	$s > a$
$f(t) = t^n$	$F(s) = \frac{n!}{s^{(n+1)}}$	$s > 0$
$f(t) = \sin(at)$	$F(s) = \frac{a}{s^2+a^2}$	$s > 0$
$f(t) = \cos(at)$	$F(s) = \frac{s}{s^2+a^2}$	$s > 0$
$f(t) = \sinh(at)$	$F(s) = \frac{a}{s^2-a^2}$	$s > \|a\|$
$f(t) = \cosh(at)$	$F(s) = \frac{s}{s^2-a^2}$	$s > \|a\|$
$f(t) = t^n e^{at}$	$F(s) = \frac{n!}{(s-a)^{(n+1)}}$	$s > \max\{a, 0\}$
$f(t) = e^{at}\sin(bt)$	$F(s) = \frac{b}{(s-a)^2+b^2}$	$s > \max\{a, 0\}$
$f(t) = e^{at}\cos(bt)$	$F(s) = \frac{(s-a)}{(s-a)^2+b^2}$	$s > \max\{a, 0\}$
$f(t) = e^{at}\sinh(bt)$	$F(s) = \frac{b}{(s-a)^2-b^2}$	$s > \max\{a, \|b\|\}$
$f(t) = e^{at}\cosh(bt)$	$F(s) = \frac{(s-a)}{(s-a)^2-b^2}$	$s > \max\{a, \|b\|\}$
$u(t-c)$	$\frac{e^{-cs}}{s}$	$s > 0,\ c \geqslant 0$
$\delta(t-c)$	e^{-cs}	$s \in \mathbb{R},\ c \geqslant 0$
$u(t-c)\,f(t-c)$	$e^{-cs}\,F(s)$	$c \geqslant 0$
$e^{ct}\,f(t)$	$F(s-c)$	$c \in \mathbb{R}$
$f'(t)$	$s\,F(s) - f(0)$	
$f''(t)$	$s^2\,F(s) - s\,f(0) - f'(0)$	
$(-t)^n\,f(t)$	$F^{(n)}(s)$	