Discrete and Continuous Models

Discrete and Continuous Models of Population Dynamics

Objectives

Students should be able to create a discrete mathematical description of a population system, then find the continuum description, solve both models, and show that the solutions are consistent.

Introduction

We study a population model with unlimited food resources and we plan to do the following:

- We construct a **discrete** mathematical models where the time variable can take only discrete values.
- We then **solve the discrete equation**, and we find solutions to our discrete models in terms of some initial population.
- We convert our discrete equation into a **continuum equation**—a differential equation—by converting the discrete time variable into a continuums time variable.
- We then **solve differential equation**, and we find solutions to our continuum models.
- Finally we verify the **consistency** of our description: The continuum limit of the solution to the discrete equation is actually the solution of the continuum equation.

Requirements

Students will need to read Sections 1.1 in both the Companion to the Lecture Notes and the Lecture Notes.

Part 1: The Discrete Model

Consider a population system with the following property: At every discrete time Δt the change in the population P between $(n+1)\Delta t$ and $n\Delta t$ is $r\Delta t$ times the population at the $n\Delta t$.

With the information above do the following:

(1.1) *(1 point)* Write the discrete equation that relates $P((n+1)\Delta t)$ with $P(n\Delta t)$.

(1.2) *(2 points)* Solve the discrete equation. Solving a discrete equation means to relate $P(n)$ with $P(0)$.

Part 2: The Continuum Model

(2.1) *(2 points)* Find the continuum limit of the discrete equation above. The continuum limit is:

$$\Delta t \to 0, \qquad \text{such that} \qquad n\,\Delta t = t. \qquad (\text{Notice that } n \to \infty.)$$

(2.2) *(2 points)* Solve the differential equation found in **(2.1)** above.

Part 3: Consistency

(3.1) *(3 points)* Show that the continuum limit of the solution to the discrete equation found in **(1.2)** is the continuum solution found in part **(2.2)**.