

# The Bernoulli Equation

*We transform a nonlinear equation into a linear equation*

## Objectives

Students should be able to identify and solve a Bernoulli equation.

## Introduction

The **Bernoulli equation** with coefficients functions  $p$ ,  $q$ , and index  $n \in \mathbb{R}$  is given by

$$y' = p(t)y + q(t)y^n. \quad (1)$$

- For  $n \neq 0, 1$  the equation is nonlinear.
- If  $n = 2$  we get the *logistic equation*.

$$y' = ry \left(1 - \frac{y}{K}\right).$$

- The Bernoulli equation in this sections is not the Bernoulli equation from fluid dynamics.
- The Bernoulli equation is special because *it is a nonlinear equation that can be transformed into a linear equation*.

## Requirements

Students will need to read the first part of section 1.5 in both the Lecture Notes Companion and the Lecture Notes. Then they need to read the last part of section 1.5 in the Lecture Notes, “The Bernoulli Equation”.

**Theorem 1.** The function  $y$  is a solution of the Bernoulli equation

$$y' = p(t)y + q(t)y^n, \quad n \neq 1,$$

iff the function  $v = 1/y^{(n-1)}$  is solution of the linear differential equation

$$v' = -(n-1)p(t)v - (n-1)q(t).$$

**Question 1.** (2 points) Prove Theorem 1.

□

**Question 2.** (*4 points*) Find every nonzero solution of the differential equation

$$y' = y + 3y^4.$$

**Question 3.** (4 points) Find every solution of the equation  $ty' = 3y + \frac{9}{2}t^5y^{1/3}$ .