Name: ______________________________ **ID Number:** ________________

TA: ______________________________ **Section Time:** ________________

Math 20D.
Final Exam
June 13, 2008

No calculators or any other devices are allowed on this exam.
Read each question carefully. If any question is not clear, ask for clarification.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
Answer each question completely, and show all your work.

1. Determine whether each of the two functions $y_1(t)$ and $y_2(t)$ given below is a solution to the given differential equation:

   (a) (10 points)

   $$y'''' + 4y''' + 3y = t, \qquad y_1(t) = e^{-t}, \quad y_2(t) = \frac{t}{3}.$$

   (b) (10 points)

   $$t^2y'' + 5ty' + 4y = 0, \quad t > 0, \qquad y_1(t) = t^{-2}, \quad y_2(t) = t^2.$$

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2. (20 points) Find the solution to the following first-order, linear, initial value problem

$$y' - 2y = e^{2t}, \qquad y(0) = 2.$$

3. (25 points) Find an explicit form of the solution to the following first-order, separable, initial value problem

$$y' = \frac{e^{-x} - e^{x}}{3 + 4y}, \qquad y(0) = 1.$$

4. (25 points) Find a solution of the following exact first-order differential equation:

$$y' = -\frac{2x + 3y}{3x + 4y}.$$

(You can leave the solution expressed in implicit form.)

5. (30 points) Find the general solution of the second-order, inhomogeneous equation

$$y'' + 6y' + 10y = 5e^{3t}.$$

(You are free to choose any method you know to solve this problem.)

6. (25 points) Find the general solution of the first-order, linear, inhomogeneous, 2×2 system of equations

$$\boldsymbol{x}'(t) = \begin{bmatrix} 1 & 4 \\ 2 & -1 \end{bmatrix} \boldsymbol{x}(t).$$

7. (25 points) Find the recurrence relation for the coefficients a_n of a power series solution $y(x) = \sum_{n=0}^{\infty} a_n x^n$ to the differential equation

$$y'' - 3y' + xy = 0.$$

(You do not need to solve the recurrence relation.)

8. (30 points) Use a Laplace transform to find the solution to the initial value problem

$$y'' + 9y = u_5(t), \qquad y(0) = 3, \quad y'(0) = 2.$$

We have denoted by $u_5(t)$ the step function

$$u_5(t) = \begin{cases} 0, & t < 5, \\ 1, & t \geqslant 5. \end{cases}$$