

Name: _____ ID Number: _____

MTH Makeup

Exam 1

September 11, 2008

No calculators or any other devices are allowed on this exam.

Read each question carefully. If any question is not clear, ask for clarification.

Write your solutions clearly and legibly; no credit will be given for illegible solutions.

Answer each question completely, and show all your work.

1. (25 points) Find the solution $y(t)$ to the initial value problem

$$t^2 y' + 3t y = \cos(t), \quad t > 0, \quad y(\pi) = 0.$$

#	Score
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2. (a) (20 points) Compute an *implicit* expression for the solution $y(x)$ to the initial value problem

$$y' = \frac{3x^2 + 4x}{2y - 2}, \quad y(0) = 2.$$

- (b) (5 points) Find the *explicit* expression for the solution found in part (2a).

3. (25 points) A tank with a capacity of 500 gallons contains at the time $t_0 = 0$ a volume $V_0 = 200$ gallons of water with $Q_0 = 100$ lb of salt dissolved in it. At a time $t_0 = 0$ minutes fresh water is pouring into the tank at a constant rate $r_i = 2$ gallons per minute, while water is also leaving the tank at a constant rate $r_o = 1$ gallons per minute with a salt concentration $q_o(t)$. Consider that there is a mixing mechanism in the tank such that the salt that enters into the tank is *instantaneously mixed* in the tank.

First, find the time t_1 when the water begins to overflow the tank. Second, find $Q(t_1)$, that is, the amount of salt in the tank at the time t_1 where the solution begins to overflow the tank.

4. (25 points) Show that the following differential equation is exact and then find an implicit expression for all solutions $y(x)$. The differential equation is the following:

$$(x^2 + y^2)(x + y y') + 2 = 0.$$