Review for Exam 4.

- 5 or 6 problems.
- Exam covers: 10.2-10.10, 11.1-11.5.
  - Infinite series (10.2).
  - The integral test (10.3).
  - Comparison tests (10.4).
  - The ratio test (10.5).
  - Alternating series (10.6).
  - Power series (10.7).
  - Taylor and Maclaurin series (10.8).
  - Convergence of Taylor series (10.9).
  - The binomial series (10.10).
  - Parametrization of plane curves (11.1).
  - Calculus with parametric curves (11.2).
  - Polar coordinates (11.3).
  - Graphing in polar coordinates (11.4).
- Areas in polar coordinates (11.5), not included.

Convergence tests for infinite series (10.2)

Example

Determine whether the series below converge or not. Specify the test you use: (a) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{2n+2}\right)$; (b) $\displaystyle\sum_{n=0}^{\infty} \frac{2^{n-1}-1}{7^n}$.

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(a) $\displaystyle\lim_{n\to\infty} \ln\left(\frac{n}{2n+2}\right) = \lim_{n\to\infty} \ln\left(\frac{n}{2n}\right)$

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The series $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{2n+2}\right)$ diverges.

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The series may converge or diverge.

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The series may converge or diverge. It converges: Geometric series.

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Solution: The series in (b) is a Geometric series.

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Solution: The series in (b) is a Geometric series. Indeed,

$$S = \sum_{n=0}^{\infty} \frac{2^{n-1}-1}{7^n}$$

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$$S = \frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{2}{7}\right)^n - \sum_{n=0}^{\infty}\left(\frac{1}{7}\right)^n$$

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$$S = \frac{1}{2}\sum_{n=0}^{\infty} \Big(\frac{2}{7}\Big)^n - \sum_{n=0}^{\infty} \Big(\frac{1}{7}\Big)^n = \frac{1}{2}\,\frac{1}{\left(1-\frac{2}{7}\right)} - \frac{1}{\left(1-\frac{1}{7}\right)}$$

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We conclude that $\displaystyle\sum_{n=0}^{\infty} \frac{2^{n-1}-1}{7^n} = \frac{7}{10} - \frac{7}{6}$. ◁

Convergence tests for infinite series (10.3)

Example

Determine whether the series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{n\sqrt{3n}}\right)$ converges or not.

Specify the test you use.

Convergence tests for infinite series (10.3)

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Determine whether the series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{n\sqrt{3n}}\right)$ converges or not.

Specify the test you use.

Solution: Notice: n-th term test trivially gives $\displaystyle\lim_{n\to\infty}\frac{2}{n\sqrt{3n}}$

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Solution: Notice: n-th term test trivially gives $\displaystyle\lim_{n\to\infty}\frac{2}{n\sqrt{3n}} = 0.$

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n-term test inconclusive.

Convergence tests for infinite series (10.3)

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Determine whether the series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{n\sqrt{3n}}\right)$ converges or not. Specify the test you use.

Solution: Notice: n-th term test trivially gives $\displaystyle\lim_{n\to\infty}\frac{2}{n\sqrt{3n}} = 0.$

n-term test inconclusive. Notice that $f(x) = \dfrac{2}{x\sqrt{3x}}$ is integrable in the interval $[1,\infty)$.

Convergence tests for infinite series (10.3)

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Determine whether the series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{n\sqrt{3n}}\right)$ converges or not.

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n-term test inconclusive. Notice that $f(x) = \dfrac{2}{x\sqrt{3x}}$ is integrable in the interval $[1,\infty)$. Indeed,

$$\int_1^{\infty}\frac{2}{x\sqrt{3x}}\,dx = \frac{2}{\sqrt{3}}\int_1^{\infty}x^{-3/2}\,dx$$

Convergence tests for infinite series (10.3)

Example

Determine whether the series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{n\sqrt{3n}}\right)$ converges or not.

Specify the test you use.

Solution: Notice: n-th term test trivially gives $\displaystyle\lim_{n\to\infty}\frac{2}{n\sqrt{3n}}=0.$

n-term test inconclusive. Notice that $f(x)=\dfrac{2}{x\sqrt{3x}}$ is integrable in the interval $[1,\infty)$. Indeed,

$$\int_1^{\infty}\frac{2}{x\sqrt{3x}}\,dx=\frac{2}{\sqrt{3}}\int_1^{\infty}x^{-3/2}\,dx=\frac{2}{\sqrt{3}}\left(-2x^{-1/2}\right)\Big|_1^{\infty}$$

Convergence tests for infinite series (10.3)

Example

Determine whether the series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{n\sqrt{3n}}\right)$ converges or not.

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Solution: Notice: n-th term test trivially gives $\displaystyle\lim_{n\to\infty}\frac{2}{n\sqrt{3n}} = 0.$

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Example

Determine whether the series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{n\sqrt{3n}}\right)$ converges or not.

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Therefore, the integral test implies that $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{n\sqrt{3n}}\right)$ converges. ◁

Convergence tests for infinite series (10.4)

Example

Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{5}{n\sqrt{n^2+8}}$ converges or not.

Specify the test you use.

Convergence tests for infinite series (10.4)

Example

Determine whether the series $\sum_{n=1}^{\infty} \frac{5}{n\sqrt{n^2+8}}$ converges or not.

Specify the test you use.

Solution: Notice: n-th term test gives $\lim_{n\to\infty} \frac{5}{n\sqrt{n^2+8}} = 0.$

Convergence tests for infinite series (10.4)

Example

Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{5}{n\sqrt{n^2+8}}$ converges or not.

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Solution: Notice: n-th term test gives $\displaystyle\lim_{n\to\infty} \frac{5}{n\sqrt{n^2+8}} = 0.$

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Convergence tests for infinite series (10.4)

Example

Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{5}{n\sqrt{n^2+8}}$ converges or not.

Specify the test you use.

Solution: Notice: n-th term test gives $\displaystyle\lim_{n\to\infty} \frac{5}{n\sqrt{n^2+8}} = 0.$

n-term test inconclusive. However we can compare the series with an $\sum(1/n^2)$ series,

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Convergence tests for infinite series (10.4)

Example

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$$n^2 < n^2 + 8$$

Convergence tests for infinite series (10.4)

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Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{5}{n\sqrt{n^2+8}}$ converges or not.

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$$n^2 < n^2 + 8 \quad \Rightarrow \quad \frac{1}{n^2+8} < \frac{1}{n^2}$$

Convergence tests for infinite series (10.4)

Example

Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{5}{n\sqrt{n^2+8}}$ converges or not.

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n-term test inconclusive. However we can compare the series with an $\sum(1/n^2)$ series, which is convergent.

$$n^2 < n^2 + 8 \quad\Rightarrow\quad \frac{1}{n^2+8} < \frac{1}{n^2} \quad\Rightarrow\quad \frac{1}{\sqrt{n^2+8}} < \frac{1}{n}$$

Convergence tests for infinite series (10.4)

Example

Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{5}{n\sqrt{n^2+8}}$ converges or not.

Specify the test you use.

Solution: Notice: n-th term test gives $\displaystyle\lim_{n\to\infty} \frac{5}{n\sqrt{n^2+8}} = 0.$

n-term test inconclusive. However we can compare the series with an $\sum(1/n^2)$ series, which is convergent.

$$n^2 < n^2 + 8 \quad \Rightarrow \quad \frac{1}{n^2+8} < \frac{1}{n^2} \quad \Rightarrow \quad \frac{1}{\sqrt{n^2+8}} < \frac{1}{n}$$

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Then comparison test implies that $\displaystyle\sum_{n=1}^{\infty} \frac{5}{n\sqrt{n^2+8}}$ converges. $\triangleleft$

Convergence tests for infinite series (10.5)

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Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(n^2+2)}{e^{6n}}$ converges or not.

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Then, the ratio test implies that $\displaystyle\sum_{n=1}^{\infty} \frac{(n^2+2)}{e^{6n}}$ converges. $\triangleleft$

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Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(n+2)!}{(-3)^n n!}$ converges or not.

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Then ratio test implies the series above converges absolutely; and the alternating series theorem implies that it converges. $\triangleleft$

Power and Taylor series (10.7-10.9)

Example

Find the Taylor polynomial order 3 centered at $x = 0$ of the function $f(x) = e^{-2x}$. Estimate the error made when using this polynomial to approximate f over $[-2, 2]$.

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Power and Taylor series (10.7-10.9)

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Graph on the *xy*-plane the curve $r(\theta) = 1 + \cos(\theta)$ for $\theta \in [0, \pi]$.
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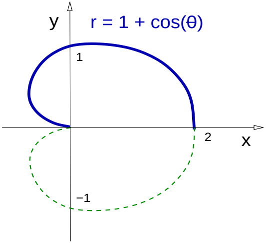

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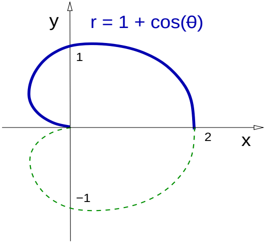

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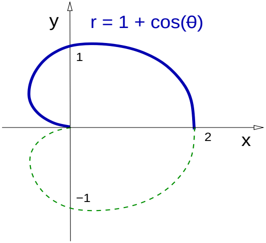

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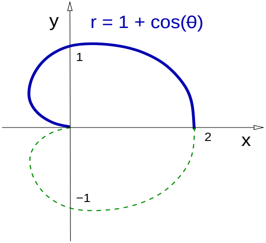

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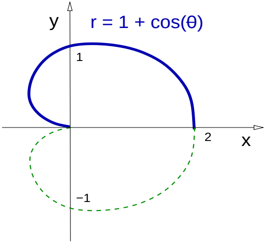

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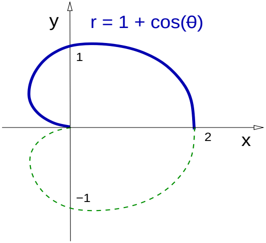

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We conclude: $m = 1$. $\triangleleft$

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Example

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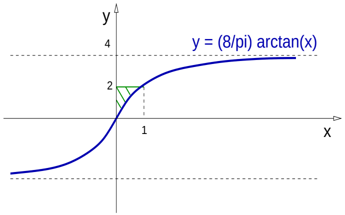

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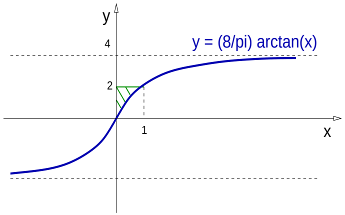


Therefore, $V = \pi \int_0^2 \big[x(y)\big]^2 \, dy$

Volumes using cross-sections (6.1)

Example

Find the volume of the region obtained by rotation the curve $x(y) = \tan(\pi y/8)$ for $y \in [0, 2]$ about the y-axis.

Solution:

To graph the function

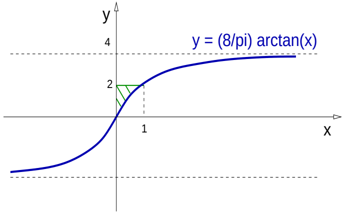
$$x = \tan(\pi y/8), \;\; y \in [0, 2],$$

one can graph

$$y = (8/\pi)\arctan(x).$$

Notice that

$$y \in [0, 2] \Rightarrow x \in [0, 1].$$



Therefore, $V = \pi \displaystyle\int_0^2 \left[x(y)\right]^2 dy = \pi \int_0^2 \left[\tan\left(\frac{\pi y}{8}\right)\right]^2 dy.$

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Volumes integrating cross-sections: General case.

Example

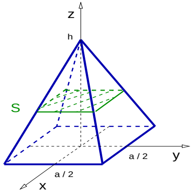
Find the volume of a pyramid with square base side a and height h.

Volumes integrating cross-sections: General case.

Example

Find the volume of a pyramid with square base side *a* and height *h*.

Solution:



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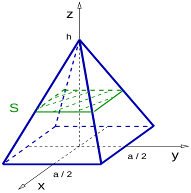

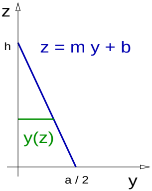

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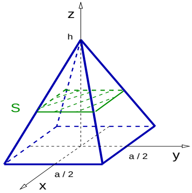

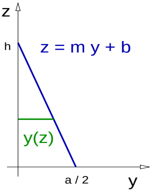

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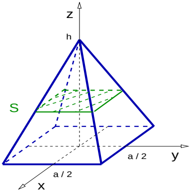

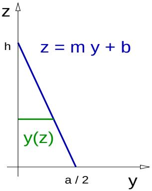

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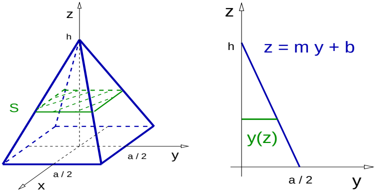

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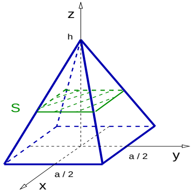

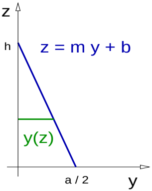

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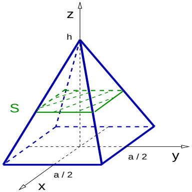

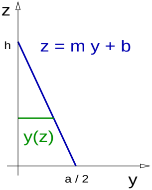

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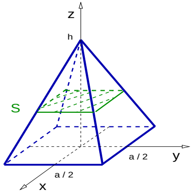

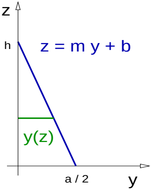

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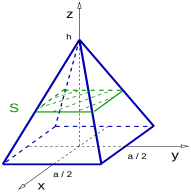

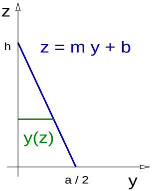

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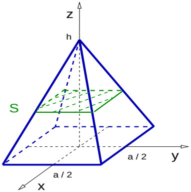

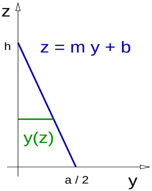

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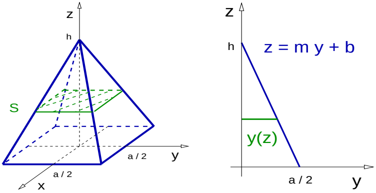

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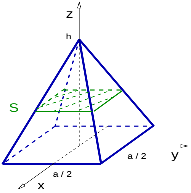

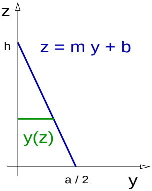

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◁

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Arc-length of curves on the plane (6.3)

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Find the arc-length of the function $y = \dfrac{x^3}{3} + \dfrac{1}{4x}$, for $x \in [1, 3]$.

Solution: Recall: $L = \displaystyle\int_{x_0}^{x_1} \sqrt{1 + \big[y'(x)\big]^2}\, dx$. Find y',

$$y'(x) = x^2 - \frac{1}{4x^2} \quad \Rightarrow \quad 1 + \big[y'(x)\big]^2 = 1 + x^4 + \frac{1}{16x^4} - \frac{1}{2},$$

$$1 + \big[y'(x)\big]^2 = x^4 + \frac{1}{16x^4} + \frac{1}{2} = \Big(x^2 + \frac{1}{4x^2}\Big)^2.$$

$$L = \int_1^3 \Big(x^2 + \frac{1}{4x^2}\Big)\, dx = \Big(\frac{x^3}{3} - \frac{1}{4x}\Big)\Big|_1^3 = 9 - \frac{1}{12} - \frac{1}{3} + \frac{1}{4}.$$

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We conclude that $L = 9 - 1/6$. ◁

Work and fluid forces: Pumping liquids

Example

A rectangular container with sides a, b, and height h, is filled with kerosene weighing $k = 51.3$ lb per cubic ft. Find the work needed to empty the container if the kerosene is pumped out from the top of the tank.

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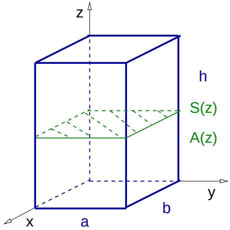
A rectangular container with sides *a*, *b*, and height *h*, is filled with kerosene weighing $k = 51.3$ lb per cubic ft. Find the work needed to empty the container if the kerosene is pumped out from the top of the tank.

Solution:



The force is the kerosene weight:

$$F = k\,A(z)\,dz$$

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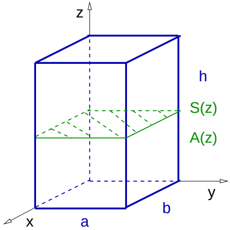

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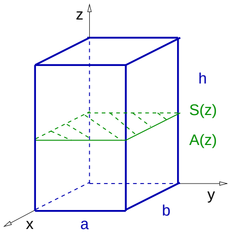

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The work done to lift that liquid from a height z to h is

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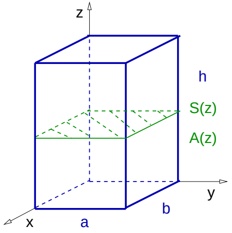

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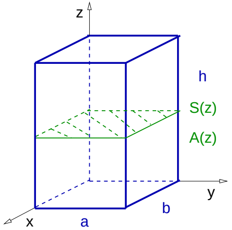

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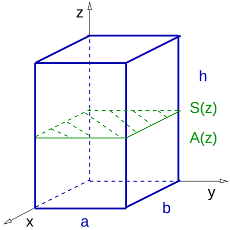

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Review for Final Exam.

- 10 or 14 problems.
- No multiple choice questions.
- No notes, no books, no calculators.
- Problems similar to homeworks.
- Exam covers:
  - Sections 6.1, 6.3, 6.5.
  - **Sections 7.1-7.7.**
  - Sections 8.1-8.5, 8.7.
  - Sections 10.1-10.10.
  - Sections 11.1-11.5.

The inverse function (7.1).

Example

Find the inverse of $f(x) = 6x^2 - 24x + 24$ for $x \leqslant 2$.

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Example

Find the inverse of $f(x) = 6x^2 - 24x + 24$ for $x \leqslant 2$.

Solution: We call $y = f(x)$, and we find $x(y)$.

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Find the inverse of $f(x) = 6x^2 - 24x + 24$ for $x \leqslant 2$.

Solution: We call $y = f(x)$, and we find $x(y)$.

$$y = 6(x^2 - 4 + 4)$$

The inverse function (7.1).

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$$y = 6(x^2 - 4 + 4) = 6(x - 2)^2$$

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Since we are interested in the inverse for $x \leqslant 2$,

$$x = 2 - \sqrt{\frac{y}{6}}.$$

The inverse function (7.1).

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Since we are interested in the inverse for $x \leqslant 2$,

$$x = 2 - \sqrt{\frac{y}{6}}.$$

We can rewrite the answer as $f^{-1}(y) = 2 - \sqrt{y/6}$.

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Since we are interested in the inverse for $x \leqslant 2$,

$$x = 2 - \sqrt{\frac{y}{6}}.$$

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It is also correct to write $f^{-1}(x) = 2 - \sqrt{x/6}$. ◁

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Example

Given $f(x) = 2x^3 + 3x^2 + 3$ for $x \geqslant 0$, find $\dfrac{df^{-1}}{dx}$ at $x = 8$.

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Example

Given $f(x) = 2x^3 + 3x^2 + 3$ for $x \geqslant 0$, find $\dfrac{df^{-1}}{dx}$ at $x = 8$.

Solution: We use y for the variable of f^{-1}.

The inverse function (7.1).

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Recall the main formula: $(f^{-1})'(y = 8) = \dfrac{1}{f'(f^{-1}(y = 8))}$.

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We need to find $x = f^{-1}(y = 8)$.

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$$8 = y$$

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by trial an error,

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Recall the main formula: $(f^{-1})'(y = 8) = \dfrac{1}{f'(f^{-1}(y = 8))}$.

We need to find $x = f^{-1}(y = 8)$. Since

$$8 = y = f(x) = 2x^3 + 3x^2 + 3,$$

by trial an error, $x = 1$.

The inverse function (7.1).

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Given $f(x) = 2x^3 + 3x^2 + 3$ for $x \geqslant 0$, find $\dfrac{df^{-1}}{dx}$ at $x = 8$.

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Recall the main formula: $(f^{-1})'(y = 8) = \dfrac{1}{f'(f^{-1}(y = 8))}$.

We need to find $x = f^{-1}(y = 8)$. Since

$$8 = y = f(x) = 2x^3 + 3x^2 + 3,$$

by trial an error, $x = 1$. So, $f(x = 1) = 8$

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Given $f(x) = 2x^3 + 3x^2 + 3$ for $x \geqslant 0$, find $\dfrac{df^{-1}}{dx}$ at $x = 8$.

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by trial an error, $x = 1$. So, $f(x = 1) = 8$ and $f^{-1}(y = 8) = 1$.

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by trial an error, $x = 1$. So, $f(x = 1) = 8$ and $f^{-1}(y = 8) = 1$.

$$(f^{-1})'(y = 8) = \frac{1}{f'(f^{-1}(y = 8))}$$

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Given $f(x) = 2x^3 + 3x^2 + 3$ for $x \geqslant 0$, find $\dfrac{df^{-1}}{dx}$ at $x = 8$.

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by trial an error, $x = 1$. So, $f(x = 1) = 8$ and $f^{-1}(y = 8) = 1$.

$$(f^{-1})'(y = 8) = \frac{1}{f'(f^{-1}(y = 8))} = \frac{1}{f'(x = 1)}.$$

We need $f'(x = 1)$.

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$$(f^{-1})'(y = 8) = \frac{1}{f'(f^{-1}(y = 8))} = \frac{1}{f'(x = 1)}.$$

We need $f'(x = 1)$. But $f'(x) = 6x^2 + 6x \;\Rightarrow\; f'(x = 1) = 12$.

We obtain $(f^{-1})'(8) = 1/12$. $\triangleleft$

The natural logarithm (7.2)

Example

Evaluate $I = \int_1^{e^{\pi}} \sin\big(\ln(x)\big)\,dx.$

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Evaluate $I = \int_1^{e^\pi} \sin\big(\ln(x)\big)\, dx$.

Solution: We try the substitution $u = \ln(x)$, hence $du = dx/x$.

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Notice $dx = x\, du = e^u\, du$. Therefore, $I = \displaystyle\int_0^{\pi} e^u \sin(u)\, du.$

The natural logarithm (7.2)

Example

Evaluate $I = \int_1^{e^\pi} \sin\big(\ln(x)\big)\, dx.$

Solution: We try the substitution $u = \ln(x)$, hence $du = dx/x$. Notice $dx = x\, du = e^u\, du$. Therefore, $I = \int_0^{\pi} e^u \sin(u)\, du.$

Integrate by parts twice,

The natural logarithm (7.2)

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Integrate by parts twice, first, $f = e^u$, $g' = \sin(u)$

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Integrate by parts twice, first, $f = e^u$, $g' = \sin(u)$

$$\int e^u \sin(u)\, du = -e^u \cos(u) + \int e^u \cos(u)\, du$$

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$$\int e^u \sin(u)\, du = -e^u \cos(u) + e^u \sin(u) - \int e^u \sin(u)\, du.$$

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Integrate by parts twice, first, $f = e^u$, $g' = \sin(u)$

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The natural logarithm (7.2)

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The natural logarithm (7.2)

Example

Simplify $f(x) = \ln\left(\frac{\sin^5(2t)}{7}\right)$, and find the derivatives of $g(x) = 3\ln(6\ln(x))$, and $h(x) = \ln\left(\sqrt{25\sin(x)\cos(x)}\right)$.

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Sometimes it is better simplify first and derivate later,

$$h(x) = \frac{1}{2}\left[\ln(25) + \ln(\sin(x)) + \ln(\cos(x)\right],$$

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We conclude that $y(x) = \sqrt[3]{1 + \sin(x)}$. ◁

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where $u_1 = \tan(\pi/3)$, and $u_0 = \tan(\pi/4)$, that is,

$$u_1 = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}, \qquad u_2 = \frac{\sqrt{2}/2}{\sqrt{2}/2}$$

Example

Evaluate $I = \displaystyle\int_{\pi/4}^{\pi/3} \frac{\sec^2(x)}{\tan(x)}\, dx.$

Solution: Substitution: $u = \tan(x)$, then $du = \sec^2(x)\, dx$.

$$I = \int_{u_0}^{u_1} \frac{du}{u} = \ln(u)\Big|_{u_0}^{u_1} = \ln(u_1) - \ln(u_0) = \ln\Big(\frac{u_1}{u_0}\Big)$$

where $u_1 = \tan(\pi/3)$, and $u_0 = \tan(\pi/4)$, that is,

$$u_1 = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}, \qquad u_2 = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1.$$

Example

Evaluate $I = \displaystyle\int_{\pi/4}^{\pi/3} \frac{\sec^2(x)}{\tan(x)}\,dx.$

Solution: Substitution: $u = \tan(x)$, then $du = \sec^2(x)\,dx$.

$$I = \int_{u_0}^{u_1} \frac{du}{u} = \ln(u)\Big|_{u_0}^{u_1} = \ln(u_1) - \ln(u_0) = \ln\left(\frac{u_1}{u_0}\right)$$

where $u_1 = \tan(\pi/3)$, and $u_0 = \tan(\pi/4)$, that is,

$$u_1 = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}, \qquad u_2 = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1.$$

We conclude: $I = \ln(\sqrt{3})$. $\triangleleft$

Sections 8.1-8.5, 8.7.

Example

Evaluate $I = \int \frac{dx}{\sqrt{x^2 - 25}}$, for $x > 5$.

Sections 8.1-8.5, 8.7.

Example

Evaluate $I = \int \frac{dx}{\sqrt{x^2 - 25}}$, for $x > 5$.

Solution: Substitution: $u = 5\sec(\theta)$,

Sections 8.1-8.5, 8.7.

Example

Evaluate $I = \int \frac{dx}{\sqrt{x^2 - 25}}$, for $x > 5$.

Solution: Substitution: $u = 5\sec(\theta)$, $du = 5\sec(\theta)\,\tan(\theta)\,d\theta$,

Sections 8.1-8.5, 8.7.

Example

Evaluate $I = \int \frac{dx}{\sqrt{x^2 - 25}}$, for $x > 5$.

Solution: Substitution: $u = 5\sec(\theta)$, $du = 5\sec(\theta)\,\tan(\theta)\,d\theta$,

$$I = \int \frac{5\sec(\theta)\,\tan(\theta)}{5\sqrt{\sec^2(\theta) - 1}}\,d\theta$$

Sections 8.1-8.5, 8.7.

Example

Evaluate $I = \int \frac{dx}{\sqrt{x^2 - 25}}$, for $x > 5$.

Solution: Substitution: $u = 5\sec(\theta)$, $du = 5\sec(\theta)\,\tan(\theta)\,d\theta$,

$$I = \int \frac{5\sec(\theta)\,\tan(\theta)}{5\sqrt{\sec^2(\theta) - 1}}\,d\theta = \int \frac{\sec(\theta)\,\tan(\theta)}{|\tan(\theta)|}\,d\theta.$$

Sections 8.1-8.5, 8.7.

Example

Evaluate $I = \int \frac{dx}{\sqrt{x^2 - 25}}$, for $x > 5$.

Solution: Substitution: $u = 5\sec(\theta)$, $du = 5\sec(\theta)\,\tan(\theta)\,d\theta$,

$$I = \int \frac{5\sec(\theta)\,\tan(\theta)}{5\sqrt{\sec^2(\theta) - 1}}\,d\theta = \int \frac{\sec(\theta)\,\tan(\theta)}{|\tan(\theta)|}\,d\theta.$$

Since $x > 5$, $\sec(\theta) > 0$

Sections 8.1-8.5, 8.7.

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Evaluate $I = \int \frac{dx}{\sqrt{x^2 - 25}}$, for $x > 5$.

Solution: Substitution: $u = 5\sec(\theta)$, $du = 5\sec(\theta)\,\tan(\theta)\,d\theta$,

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Since $x > 5$, $\sec(\theta) > 0$ so $\tan(\theta) > 0$.

Sections 8.1-8.5, 8.7.

Example

Evaluate $I = \displaystyle\int \frac{dx}{\sqrt{x^2 - 25}}$, for $x > 5$.

Solution: Substitution: $u = 5\sec(\theta)$, $du = 5\sec(\theta)\,\tan(\theta)\,d\theta$,

$$I = \int \frac{5\sec(\theta)\,\tan(\theta)}{5\sqrt{\sec^2(\theta) - 1}}\,d\theta = \int \frac{\sec(\theta)\,\tan(\theta)}{|\tan(\theta)|}\,d\theta.$$

Since $x > 5$, $\sec(\theta) > 0$ so $\tan(\theta) > 0$.

$$I = \int \frac{\sec(\theta)\,\tan(\theta)}{\tan(\theta)}\,d\theta$$

Sections 8.1-8.5, 8.7.

Example

Evaluate $I = \int \frac{dx}{\sqrt{x^2 - 25}}$, for $x > 5$.

Solution: Substitution: $u = 5\sec(\theta)$, $du = 5\sec(\theta)\,\tan(\theta)\,d\theta$,

$$I = \int \frac{5\sec(\theta)\,\tan(\theta)}{5\sqrt{\sec^2(\theta) - 1}}\,d\theta = \int \frac{\sec(\theta)\,\tan(\theta)}{|\tan(\theta)|}\,d\theta.$$

Since $x > 5$, $\sec(\theta) > 0$ so $\tan(\theta) > 0$.

$$I = \int \frac{\sec(\theta)\,\tan(\theta)}{\tan(\theta)}\,d\theta = \int \sec(\theta)\,d\theta.$$

Sections 8.1-8.5, 8.7.

Example

Evaluate $I = \int \frac{dx}{\sqrt{x^2 - 25}}$, for $x > 5$.

Solution: Substitution: $u = 5\sec(\theta)$, $du = 5\sec(\theta)\,\tan(\theta)\,d\theta$,

$$I = \int \frac{5\sec(\theta)\,\tan(\theta)}{5\sqrt{\sec^2(\theta) - 1}}\,d\theta = \int \frac{\sec(\theta)\,\tan(\theta)}{|\tan(\theta)|}\,d\theta.$$

Since $x > 5$, $\sec(\theta) > 0$ so $\tan(\theta) > 0$.

$$I = \int \frac{\sec(\theta)\,\tan(\theta)}{\tan(\theta)}\,d\theta = \int \sec(\theta)\,d\theta.$$

$$I = \int \sec(\theta)\,\frac{\big(\sec(\theta) + \tan(\theta)\big)}{\big(\sec(\theta) + \tan(\theta)\big)}\,d\theta.$$

Sections 8.1-8.5, 8.7.

Example

Evaluate $I = \int \frac{dx}{\sqrt{x^2 - 25}}$, for $x > 5$.

Solution: Recall: $I = \int \sec(\theta) \frac{\big(\sec(\theta) + \tan(\theta)\big)}{\big(\sec(\theta) + \tan(\theta)\big)} \, d\theta.$

Sections 8.1-8.5, 8.7.

Example

Evaluate $I = \int \frac{dx}{\sqrt{x^2 - 25}}$, for $x > 5$.

Solution: Recall: $I = \int \sec(\theta)\, \frac{\big(\sec(\theta) + \tan(\theta)\big)}{\big(\sec(\theta) + \tan(\theta)\big)}\, d\theta.$

$$I = \int \frac{\big(\sec^2(\theta) + \sec(\theta)\, \tan(\theta)\big)}{\big(\sec(\theta) + \tan(\theta)\big)}\, d\theta$$

Example

Evaluate $I = \int \frac{dx}{\sqrt{x^2 - 25}}$, for $x > 5$.

Solution: Recall: $I = \int \sec(\theta) \, \frac{\big(\sec(\theta) + \tan(\theta)\big)}{\big(\sec(\theta) + \tan(\theta)\big)} \, d\theta.$

$$I = \int \frac{\big(\sec^2(\theta) + \sec(\theta)\, \tan(\theta)\big)}{\big(\sec(\theta) + \tan(\theta)\big)} \, d\theta = \int \frac{\big(\tan'(\theta) + \sec'(\theta)\big)}{\big(\sec(\theta) + \tan(\theta)\big)} \, d\theta.$$

Sections 8.1-8.5, 8.7.

Example

Evaluate $I = \displaystyle\int \frac{dx}{\sqrt{x^2 - 25}}$, for $x > 5$.

Solution: Recall: $I = \displaystyle\int \sec(\theta)\,\frac{\big(\sec(\theta) + \tan(\theta)\big)}{\big(\sec(\theta) + \tan(\theta)\big)}\,d\theta.$

$$I = \int \frac{\big(\sec^2(\theta) + \sec(\theta)\,\tan(\theta)\big)}{\big(\sec(\theta) + \tan(\theta)\big)}\,d\theta = \int \frac{\big(\tan'(\theta) + \sec'(\theta)\big)}{\big(\sec(\theta) + \tan(\theta)\big)}\,d\theta.$$

Substitute $u = \sec(\theta) + \tan(\theta)$,

Sections 8.1-8.5, 8.7.

Example

Evaluate $I = \int \frac{dx}{\sqrt{x^2 - 25}}$, for $x > 5$.

Solution: Recall: $I = \int \sec(\theta) \, \frac{\big(\sec(\theta) + \tan(\theta)\big)}{\big(\sec(\theta) + \tan(\theta)\big)} \, d\theta.$

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Substitute $u = \sec(\theta) + \tan(\theta)$, then $I = \int \frac{du}{u}$

Sections 8.1-8.5, 8.7.

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Evaluate $I = \int \frac{dx}{\sqrt{x^2 - 25}}$, for $x > 5$.

Solution: Recall: $I = \int \sec(\theta)\, \frac{\big(\sec(\theta) + \tan(\theta)\big)}{\big(\sec(\theta) + \tan(\theta)\big)}\, d\theta.$

$$I = \int \frac{\big(\sec^2(\theta) + \sec(\theta)\, \tan(\theta)\big)}{\big(\sec(\theta) + \tan(\theta)\big)}\, d\theta = \int \frac{\big(\tan'(\theta) + \sec'(\theta)\big)}{\big(\sec(\theta) + \tan(\theta)\big)}\, d\theta.$$

Substitute $u = \sec(\theta) + \tan(\theta)$, then $I = \int \frac{du}{u} = \ln(u) + c.$

Sections 8.1-8.5, 8.7.

Example

Evaluate $I = \int \frac{dx}{\sqrt{x^2 - 25}}$, for $x > 5$.

Solution: Recall: $I = \int \sec(\theta) \frac{\big(\sec(\theta) + \tan(\theta)\big)}{\big(\sec(\theta) + \tan(\theta)\big)} \, d\theta.$

$$I = \int \frac{\big(\sec^2(\theta) + \sec(\theta)\, \tan(\theta)\big)}{\big(\sec(\theta) + \tan(\theta)\big)} \, d\theta = \int \frac{\big(\tan'(\theta) + \sec'(\theta)\big)}{\big(\sec(\theta) + \tan(\theta)\big)} \, d\theta.$$

Substitute $u = \sec(\theta) + \tan(\theta)$, then $I = \int \frac{du}{u} = \ln(u) + c$.

$I = \ln\big(\sec(\theta) + \tan(\theta)\big) + c,$

Sections 8.1-8.5, 8.7.

Example

Evaluate $I = \displaystyle\int \frac{dx}{\sqrt{x^2 - 25}}$, for $x > 5$.

Solution: Recall: $I = \displaystyle\int \sec(\theta)\, \frac{\big(\sec(\theta) + \tan(\theta)\big)}{\big(\sec(\theta) + \tan(\theta)\big)}\, d\theta.$

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Substitute $u = \sec(\theta) + \tan(\theta)$, then $I = \displaystyle\int \frac{du}{u} = \ln(u) + c.$

$I = \ln\big(\sec(\theta) + \tan(\theta)\big) + c, \quad \sec(\theta) = \dfrac{x}{5},$

Sections 8.1-8.5, 8.7.

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Evaluate $I = \int \frac{dx}{\sqrt{x^2 - 25}}$, for $x > 5$.

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Substitute $u = \sec(\theta) + \tan(\theta)$, then $I = \int \frac{du}{u} = \ln(u) + c.$

$I = \ln\big(\sec(\theta) + \tan(\theta)\big) + c, \quad \sec(\theta) = \frac{x}{5}, \quad \tan^2(\theta) + 1 = \sec^2(\theta).$

Sections 8.1-8.5, 8.7.

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Evaluate $I = \int \frac{dx}{\sqrt{x^2 - 25}}$, for $x > 5$.

Solution: Recall: $I = \int \sec(\theta) \frac{\big(\sec(\theta) + \tan(\theta)\big)}{\big(\sec(\theta) + \tan(\theta)\big)} \, d\theta$.

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Substitute $u = \sec(\theta) + \tan(\theta)$, then $I = \int \frac{du}{u} = \ln(u) + c$.

$I = \ln\big(\sec(\theta) + \tan(\theta)\big) + c, \quad \sec(\theta) = \frac{x}{5}, \quad \tan^2(\theta) + 1 = \sec^2(\theta)$.

$$I = \ln\left(\frac{x}{5} + \sqrt{\frac{x^2}{5^2} - 1}\right) + c. \qquad \triangleleft$$

Improper integrals (8.7): Comparison tests

Example

Determine whether $I = \int_3^{\infty} \frac{x\,dx}{\sqrt[3]{x^5 + x^3}}$ converges or not.

Improper integrals (8.7): Comparison tests

Example

Determine whether $I = \int_3^\infty \frac{x\,dx}{\sqrt[3]{x^5 + x^3}}$ converges or not.

Solution: Limit comparison test.

Improper integrals (8.7): Comparison tests

Example

Determine whether $I = \int_3^{\infty} \frac{x\,dx}{\sqrt[3]{x^5 + x^3}}$ converges or not.

Solution: Limit comparison test. Let $g(x)$ such that:

Improper integrals (8.7): Comparison tests

Example

Determine whether $I = \int_3^\infty \frac{x\,dx}{\sqrt[3]{x^5 + x^3}}$ converges or not.

Solution: Limit comparison test. Let $g(x)$ such that:

$$\lim_{x\to\infty} g(x) = \lim_{x\to\infty} \frac{x}{\sqrt[3]{x^5 + x^3}}$$

Improper integrals (8.7): Comparison tests

Example

Determine whether $I = \int_3^{\infty} \frac{x\,dx}{\sqrt[3]{x^5 + x^3}}$ converges or not.

Solution: Limit comparison test. Let $g(x)$ such that:

$$\lim_{x\to\infty} g(x) = \lim_{x\to\infty} \frac{x}{\sqrt[3]{x^5 + x^3}} = \lim_{x\to\infty} \frac{1}{x^{\frac{5}{3}-1}}$$

Improper integrals (8.7): Comparison tests

Example

Determine whether $I = \int_3^\infty \frac{x\,dx}{\sqrt[3]{x^5 + x^3}}$ converges or not.

Solution: Limit comparison test. Let $g(x)$ such that:

$$\lim_{x\to\infty} g(x) = \lim_{x\to\infty} \frac{x}{\sqrt[3]{x^5 + x^3}} = \lim_{x\to\infty} \frac{1}{x^{\frac{5}{3}-1}} = \lim_{x\to\infty} \frac{1}{x^{\frac{2}{3}}}.$$

Improper integrals (8.7): Comparison tests

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Therefore, we use the limit comparison test with $g(x) = \frac{1}{x^{2/3}}$.

Improper integrals (8.7): Comparison tests

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Therefore, we use the limit comparison test with $g(x) = \frac{1}{x^{2/3}}$.

Then, by construction,

Improper integrals (8.7): Comparison tests

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Therefore, we use the limit comparison test with $g(x) = \frac{1}{x^{2/3}}$.

Then, by construction,

$$\lim_{x\to\infty} \left(\frac{x}{\sqrt[3]{x^5 + x^3}} \right) \left(x^{2/3} \right)$$

Improper integrals (8.7): Comparison tests

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Solution: Limit comparison test. Let $g(x)$ such that:

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Therefore, we use the limit comparison test with $g(x) = \frac{1}{x^{2/3}}$.

Then, by construction,

$$\lim_{x\to\infty} \left(\frac{x}{\sqrt[3]{x^5 + x^3}}\right)\left(x^{2/3}\right) = \lim_{x\to\infty} \left(\frac{1}{x^{\frac{5}{3}-1}}\right)\left(x^{2/3}\right)$$

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Determine whether $I = \int_3^\infty \frac{x\,dx}{\sqrt[3]{x^5 + x^3}}$ converges or not.

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Therefore, we use the limit comparison test with $g(x) = \frac{1}{x^{2/3}}$.
Then, by construction,

$$\lim_{x\to\infty} \left(\frac{x}{\sqrt[3]{x^5 + x^3}}\right)(x^{2/3}) = \lim_{x\to\infty} \left(\frac{1}{x^{\frac{5}{3}-1}}\right)(x^{2/3}) = 1.$$

Improper integrals (8.7): Comparison tests

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Determine whether $I = \int_3^{\infty} \frac{x\,dx}{\sqrt[3]{x^5 + x^3}}$ converges or not.

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Therefore, we use the limit comparison test with $g(x) = \frac{1}{x^{2/3}}$.

Then, by construction,

$$\lim_{x\to\infty} \Big(\frac{x}{\sqrt[3]{x^5 + x^3}}\Big)\big(x^{2/3}\big) = \lim_{x\to\infty} \Big(\frac{1}{x^{\frac{5}{3}-1}}\Big)\big(x^{2/3}\big) = 1.$$

Since $\int_3^{\infty} x^{-2/3}\,dx = 3\,x^{1/3}\Big|_3^{\infty}$

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Determine whether $I = \int_3^\infty \frac{x\,dx}{\sqrt[3]{x^5 + x^3}}$ converges or not.

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Therefore, we use the limit comparison test with $g(x) = \frac{1}{x^{2/3}}$.

Then, by construction,

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Since $\int_3^\infty x^{-2/3}\,dx = 3\,x^{1/3}\Big|_3^\infty$ diverges,

Improper integrals (8.7): Comparison tests

Example

Determine whether $I = \int_3^\infty \frac{x\,dx}{\sqrt[3]{x^5 + x^3}}$ converges or not.

Solution: Limit comparison test. Let $g(x)$ such that:

$$\lim_{x\to\infty} g(x) = \lim_{x\to\infty} \frac{x}{\sqrt[3]{x^5 + x^3}} = \lim_{x\to\infty} \frac{1}{x^{\frac{5}{3}-1}} = \lim_{x\to\infty} \frac{1}{x^{\frac{2}{3}}}.$$

Therefore, we use the limit comparison test with $g(x) = \frac{1}{x^{2/3}}$.
Then, by construction,

$$\lim_{x\to\infty} \left(\frac{x}{\sqrt[3]{x^5 + x^3}}\right)\left(x^{2/3}\right) = \lim_{x\to\infty} \left(\frac{1}{x^{\frac{5}{3}-1}}\right)\left(x^{2/3}\right) = 1.$$

Since $\int_3^\infty x^{-2/3}\,dx = 3\,x^{1/3}\Big|_3^\infty$ diverges, then I diverges too. ◁

Review for Final Exam.

- 10 or 14 problems.
- No multiple choice questions.
- No notes, no books, no calculators.
- Problems similar to homeworks.
- Exam covers:
  - Sections 6.1, 6.3, 6.5.
  - Sections 7.1-7.7.
  - Sections 8.1-8.5, 8.7.
  - **Sections 10.1-10.10.**
  - Sections 11.1-11.5.

Infinite series (10.2)

Example

Determine whether the series $\sum_{n=1}^{\infty} \left(\frac{2}{n}\right)^{\frac{3}{n}}$ converges or not.

Infinite series (10.2)

Example

Determine whether the series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{n}\right)^{\frac{3}{n}}$ converges or not.

Solution: The n-term test.

Infinite series (10.2)

Example

Determine whether the series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{n}\right)^{\frac{3}{n}}$ converges or not.

Solution: The n-term test. We will need L'Hôpital's rule.

Infinite series (10.2)

Example

Determine whether the series $\sum_{n=1}^{\infty} \left(\frac{2}{n}\right)^{\frac{3}{n}}$ converges or not.

Solution: The n-term test. We will need L'Hôpital's rule. Introduce the function $f(x) = \left(\frac{2}{x}\right)^{\frac{3}{x}}$.

Infinite series (10.2)

Example

Determine whether the series $\sum_{n=1}^{\infty} \left(\frac{2}{n}\right)^{\frac{3}{n}}$ converges or not.

Solution: The n-term test. We will need L'Hôpital's rule. Introduce the function $f(x) = \left(\frac{2}{x}\right)^{\frac{3}{x}}$.

$$\lim_{x\to\infty} \left(\frac{2}{x}\right)^{\frac{3}{x}}$$

Infinite series (10.2)

Example

Determine whether the series $\sum_{n=1}^{\infty} \left(\frac{2}{n}\right)^{\frac{3}{n}}$ converges or not.

Solution: The *n*-term test. We will need L'Hôpital's rule. Introduce the function $f(x) = \left(\frac{2}{x}\right)^{\frac{3}{x}}$.

$$\lim_{x \to \infty} \left(\frac{2}{x}\right)^{\frac{3}{x}} = \lim_{x \to \infty} e^{\left[\frac{3 \ln\left(\frac{2}{x}\right)}{x}\right]}$$

Infinite series (10.2)

Example

Determine whether the series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{n}\right)^{\frac{3}{n}}$ converges or not.

Solution: The n-term test. We will need L'Hôpital's rule. Introduce the function $f(x) = \left(\frac{2}{x}\right)^{\frac{3}{x}}$.

$$\lim_{x\to\infty}\left(\frac{2}{x}\right)^{\frac{3}{x}} = \lim_{x\to\infty} e^{\left[\frac{3\ln\left(\frac{2}{x}\right)}{x}\right]} = e^{3\lim_{x\to\infty}\left[\frac{\ln\left(\frac{2}{x}\right)}{x}\right]}$$

Infinite series (10.2)

Example

Determine whether the series $\sum_{n=1}^{\infty} \left(\frac{2}{n}\right)^{\frac{3}{n}}$ converges or not.

Solution: The n-term test. We will need L'Hôpital's rule. Introduce the function $f(x) = \left(\frac{2}{x}\right)^{\frac{3}{x}}$.

$$\lim_{x\to\infty} \left(\frac{2}{x}\right)^{\frac{3}{x}} = \lim_{x\to\infty} e^{\left[\frac{3\ln\left(\frac{2}{x}\right)}{x}\right]} = e^{3\lim_{x\to\infty}\left[\frac{\ln\left(\frac{2}{x}\right)}{x}\right]}$$

L'Hôpital's rule to find the limit in the exponent;

Infinite series (10.2)

Example

Determine whether the series $\sum_{n=1}^{\infty}\left(\frac{2}{n}\right)^{\frac{3}{n}}$ converges or not.

Solution: The n-term test. We will need L'Hôpital's rule. Introduce the function $f(x) = \left(\frac{2}{x}\right)^{\frac{3}{x}}$.

$$\lim_{x\to\infty}\left(\frac{2}{x}\right)^{\frac{3}{x}} = \lim_{x\to\infty} e^{\left[\frac{3\ln\left(\frac{2}{x}\right)}{x}\right]} = e^{3\lim_{x\to\infty}\left[\frac{\ln\left(\frac{2}{x}\right)}{x}\right]}$$

L'Hôpital's rule to find the limit in the exponent;

$$\tilde{L} = \lim_{x\to\infty}\frac{\ln\left(\frac{2}{x}\right)}{x}$$

Infinite series (10.2)

Example

Determine whether the series $\sum_{n=1}^{\infty} \left(\frac{2}{n}\right)^{\frac{3}{n}}$ converges or not.

Solution: The n-term test. We will need L'Hôpital's rule. Introduce the function $f(x) = \left(\frac{2}{x}\right)^{\frac{3}{x}}$.

$$\lim_{x\to\infty} \left(\frac{2}{x}\right)^{\frac{3}{x}} = \lim_{x\to\infty} e^{\left[\frac{3\ln\left(\frac{2}{x}\right)}{x}\right]} = e^{3\lim_{x\to\infty}\left[\frac{\ln\left(\frac{2}{x}\right)}{x}\right]}$$

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$\lim_{x\to\infty} f(x) = e^0$

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$\lim_{x\to\infty} f(x) = e^0 = 1$, then $\displaystyle\lim_{n\to\infty}\left(\frac{2}{n}\right)^{\frac{3}{n}} = 1$. The series diverges.

Convergence tests for infinite series (10.5)

Example

Determine whether the series $\sum_{n=1}^{\infty} \frac{(n!)}{(n+1)\, e^n}$ converges or not.

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Therefore, $\displaystyle\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$

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Therefore, $\displaystyle\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)}{e} = \infty.$

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Therefore, $\displaystyle\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)}{e} = \infty.$

Then, the ratio test implies that $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)}{(n+1)\, e^n}$ diverges. $\triangleleft$

Power and Taylor series (10.7-10.9)

Example

Find the T_3 centered at $x = 0$ of $f(x) = \dfrac{1}{1-x}$ and estimate the error of using T_3 to approximate f over $[-1/2, 1/2]$.

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Review for Final Exam.

- 10 or 14 problems.
- No multiple choice questions.
- No notes, no books, no calculators.
- Problems similar to homeworks.
- Exam covers:
  - Sections 6.1, 6.3, 6.5.
  - Sections 7.1-7.7.
  - Sections 8.1-8.5, 8.7.
  - Sections 10.1-10.10.
  - **Sections 11.1-11.5.**

Parametric curves and polar coordinates (11.1-11.4)

Example

Compute the area shared by the interiors of the curves $r_1(\theta) = 1 + \cos(\theta)$ and $r_2(\theta) = 1 - \cos(\theta)$ for $\theta \in [0, 2\pi]$.

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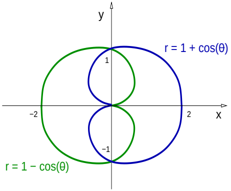

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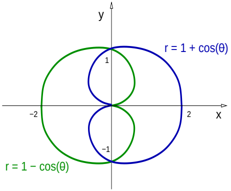

By symmetry, the area of the interiors is

$$A = 4 \int_0^{\pi/2} \frac{1}{2} \left(1 - \cos(\theta)\right)^2 d\theta$$

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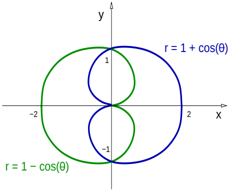

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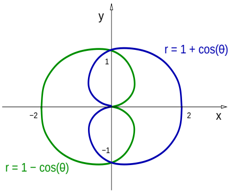

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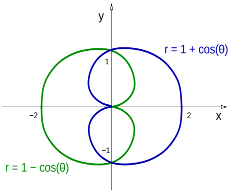
$$A = 2\Big[\frac{\pi}{2} - 2\sin(\theta)\Big|_0^{\pi/2} + 2 \int_0^{\pi/2} \frac{1}{2} \big(1 + \cos(2\theta)\big) \, d\theta.$$

Parametric curves and polar coordinates (11.1-11.4)

Example

Compute the area shared by the interiors of the curves $r_1(\theta) = 1 + \cos(\theta)$ and $r_2(\theta) = 1 - \cos(\theta)$ for $\theta \in [0, 2\pi]$.

Solution: This is the intersection of two cardiods



By symmetry, the area of the interiors is

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$$A = \pi - 4 + \frac{\pi}{2} + \frac{1}{2}\sin(2\theta)\Big|_0^{\pi/2}$$

Parametric curves and polar coordinates (11.1-11.4)

Example

Compute the area shared by the interiors of the curves $r_1(\theta) = 1 + \cos(\theta)$ and $r_2(\theta) = 1 - \cos(\theta)$ for $\theta \in [0, 2\pi]$.

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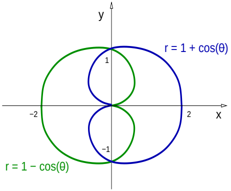

By symmetry, the area of the interiors is

$$A = 4 \int_0^{\pi/2} \frac{1}{2} \big(1 - \cos(\theta)\big)^2 \, d\theta$$

$$A = 2 \int_0^{\pi/2} \big(1 - 2\cos(\theta) + \cos^2(\theta)\big) \, d\theta$$

$$A = 2\Big[\frac{\pi}{2} - 2\sin(\theta)\Big|_0^{\pi/2} + 2 \int_0^{\pi/2} \frac{1}{2} \big(1 + \cos(2\theta)\big) \, d\theta.$$

$$A = \pi - 4 + \frac{\pi}{2} + \frac{1}{2} \sin(2\theta)\Big|_0^{\pi/2} \quad \Rightarrow \quad A = \frac{3\pi}{2} - 4. \qquad \lhd$$