Review for Exam 3.

- \blacktriangleright 5 or 6 problems.
- \triangleright No multiple choice questions.
- \triangleright No notes, no books, no calculators.
- \blacktriangleright Problems similar to homeworks.
- Exam covers: 8.3, 8.4, 7.5, 8.7, 10.1.
	- \blacktriangleright Trigonometric substitutions (8.3).
	- Integration using partial fractions (8.4) .

AD A REAKEN E VAN

- \blacktriangleright L'Hôpital's rule (7.5).
- \blacktriangleright Improper integrals (8.7).
- Infinite sequences (10.1) .
- \blacktriangleright Section not covered:
	- Integration using tables (8.5) .

Review for Exam 3.

- \blacktriangleright 5 or 6 problems.
- \triangleright No multiple choice questions.
- \triangleright No notes, no books, no calculators.
- \blacktriangleright Problems similar to homeworks.
- Exam covers: 8.3, 8.4, 7.5, 8.7, 10.1.
	- \triangleright Trigonometric substitutions (8.3).
	- Integration using partial fractions (8.4) .

AD A REAKEN E VAN

- \blacktriangleright L'Hôpital's rule (7.5).
- \blacktriangleright Improper integrals (8.7).
- Infinite sequences (10.1) .
- \blacktriangleright Section not covered:
	- Integration using tables (8.5) .

Recall: From Sect. 8.2: $\int \sec(\theta) d\theta = \ln(\sec(\theta) + \tan(\theta)) + c$.

Example

Evaluate $I = \int \frac{e^x dx}{\sqrt{2x}}$ $e^{2x} + 9$. Express you result in terms of the variable x.

Recall: From Sect. 8.2:
$$
\int \sec(\theta) d\theta = \ln(\sec(\theta) + \tan(\theta)) + c
$$
.

Example

Evaluate $I = \int \frac{e^x dx}{\sqrt{2x}}$ $e^{2x} + 9$. Express you result in terms of the variable x.

K ロ ▶ K @ ▶ K 할 X K 할 X 시 할 X 10 Q Q Q

Solution: First substitution,

Recall: From Sect. 8.2:
$$
\int \sec(\theta) d\theta = \ln(\sec(\theta) + \tan(\theta)) + c
$$
.

Example

Evaluate $I = \int \frac{e^x dx}{\sqrt{2x}}$ $e^{2x} + 9$. Express you result in terms of the variable x.

K ロ ▶ K @ ▶ K 할 X K 할 X 시 할 X 10 Q Q Q

Solution: First substitution, $y = e^x$,

Recall: From Sect. 8.2:
$$
\int \sec(\theta) d\theta = \ln(\sec(\theta) + \tan(\theta)) + c
$$
.

Example

Evaluate $I = \int \frac{e^x dx}{\sqrt{2x}}$ $e^{2x} + 9$. Express you result in terms of the variable x.

K ロ ▶ K @ ▶ K 할 X K 할 X 시 할 X 10 Q Q Q

Solution: First substitution, $y = e^x$, then $dy = e^x dx$,

Recall: From Sect. 8.2: $\int \sec(\theta) d\theta = \ln(\sec(\theta) + \tan(\theta)) + c$.

Example

Evaluate $I = \int \frac{e^x dx}{\sqrt{2x}}$ $e^{2x} + 9$. Express you result in terms of the variable x.

Solution: First substitution, $y = e^x$, then $dy = e^x dx$, $y > 0$,

Recall: From Sect. 8.2:
$$
\int \sec(\theta) d\theta = \ln(\sec(\theta) + \tan(\theta)) + c
$$
.

Example

Evaluate $I = \int \frac{e^x dx}{\sqrt{2x}}$ $e^{2x} + 9$. Express you result in terms of the variable x.

Solution: First substitution, $y = e^x$, then $dy = e^x dx$, $y > 0$,

$$
I = \int \frac{dy}{\sqrt{y^2 + 9}}.
$$

Recall: From Sect. 8.2:
$$
\int \sec(\theta) d\theta = \ln(\sec(\theta) + \tan(\theta)) + c
$$
.

Example

Evaluate $I = \int \frac{e^x dx}{\sqrt{2x}}$ $e^{2x} + 9$. Express you result in terms of the variable x.

Solution: First substitution, $y = e^x$, then $dy = e^x dx$, $y > 0$,

$$
I = \int \frac{dy}{\sqrt{y^2 + 9}}
$$
. Second subs.:

Recall: From Sect. 8.2:
$$
\int \sec(\theta) d\theta = \ln(\sec(\theta) + \tan(\theta)) + c
$$
.

Example

Evaluate $I = \int \frac{e^x dx}{\sqrt{2x}}$ $e^{2x} + 9$. Express you result in terms of the variable x.

Solution: First substitution, $y = e^x$, then $dy = e^x dx$, $y > 0$,

$$
I = \int \frac{dy}{\sqrt{y^2 + 9}}
$$
. Second subs.:
$$
\begin{cases} y = 3 \tan(\theta), \\ dy = 3 \sec^2(\theta) d\theta, \\ \theta \in (0, \pi/2). \end{cases}
$$

Recall: From Sect. 8.2:
$$
\int \sec(\theta) d\theta = \ln(\sec(\theta) + \tan(\theta)) + c
$$
.

 \overline{a}

Example

Evaluate $I = \int \frac{e^x dx}{\sqrt{2x}}$ $e^{2x} + 9$. Express you result in terms of the variable x.

Solution: First substitution, $y = e^x$, then $dy = e^x dx$, $y > 0$,

$$
I = \int \frac{dy}{\sqrt{y^2 + 9}}
$$
. Second subs.:
$$
\begin{cases} y = 3 \tan(\theta), \\ dy = 3 \sec^2(\theta) d\theta, \\ \theta \in (0, \pi/2). \end{cases}
$$

$$
I = \int \frac{3\sec^2(\theta)\,d\theta}{\sqrt{9\tan^2(\theta)+9}}
$$

Recall: From Sect. 8.2:
$$
\int \sec(\theta) d\theta = \ln(\sec(\theta) + \tan(\theta)) + c
$$
.

 \overline{a}

Example

Evaluate $I = \int \frac{e^x dx}{\sqrt{2x}}$ $e^{2x} + 9$. Express you result in terms of the variable x.

Solution: First substitution, $y = e^x$, then $dy = e^x dx$, $y > 0$,

$$
I = \int \frac{dy}{\sqrt{y^2 + 9}}
$$
. Second subs.:
$$
\begin{cases} y = 3 \tan(\theta), \\ dy = 3 \sec^2(\theta) d\theta, \\ \theta \in (0, \pi/2). \end{cases}
$$

$$
I = \int \frac{3\sec^2(\theta)\,d\theta}{\sqrt{9\tan^2(\theta) + 9}} = \int \frac{3\sec^2(\theta)\,d\theta}{3\sqrt{\tan^2(\theta) + 1}}
$$

Recall: From Sect. 8.2:
$$
\int \sec(\theta) d\theta = \ln(\sec(\theta) + \tan(\theta)) + c
$$
.

 \overline{a}

Example

Evaluate $I = \int \frac{e^x dx}{\sqrt{2x}}$ $e^{2x} + 9$. Express you result in terms of the variable x.

Solution: First substitution, $y = e^x$, then $dy = e^x dx$, $y > 0$,

$$
I = \int \frac{dy}{\sqrt{y^2 + 9}}
$$
. Second subs.:
$$
\begin{cases} y = 3 \tan(\theta), \\ dy = 3 \sec^2(\theta) d\theta, \\ \theta \in (0, \pi/2). \end{cases}
$$

$$
I = \int \frac{3\sec^2(\theta)\,d\theta}{\sqrt{9\tan^2(\theta)+9}} = \int \frac{3\sec^2(\theta)\,d\theta}{3\sqrt{\tan^2(\theta)+1}} = \int \frac{\sec^2(\theta)\,d\theta}{|\sec(\theta)|}.
$$

Example Evaluate $I = \int \frac{e^{x} dx}{\sqrt{2x}}$ $e^{2x} + 9$. Express you result in terms of the variable x.

Solution: So:
$$
I = \int \frac{\sec^2(\theta) d\theta}{|\sec(\theta)|}
$$
; $e^x = y = 3\tan(\theta)$; $\theta \in \left(0, \frac{\pi}{2}\right)$.

Example Evaluate $I = \int \frac{e^{x} dx}{\sqrt{2x}}$ $e^{2x} + 9$. Express you result in terms of the variable x.

Solution: So:
$$
I = \int \frac{\sec^2(\theta) d\theta}{|\sec(\theta)|}
$$
; $e^x = y = 3 \tan(\theta)$; $\theta \in (0, \frac{\pi}{2})$.
 $I = \int \sec(\theta) d\theta$

Example Evaluate $I = \int \frac{e^{x} dx}{\sqrt{2x}}$ $e^{2x} + 9$. Express you result in terms of the variable x.

Solution: So:
$$
I = \int \frac{\sec^2(\theta) d\theta}{|\sec(\theta)|}
$$
; $e^x = y = 3 \tan(\theta)$; $\theta \in (0, \frac{\pi}{2})$.

$$
I = \int \sec(\theta) d\theta = \ln(\sec(\theta) + \tan(\theta)) + c
$$

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

Example Evaluate $I = \int \frac{e^{x} dx}{\sqrt{2x}}$ $e^{2x} + 9$. Express you result in terms of the variable x.

Solution: So:
$$
I = \int \frac{\sec^2(\theta) d\theta}{|\sec(\theta)|}
$$
; $e^x = y = 3 \tan(\theta)$; $\theta \in (0, \frac{\pi}{2})$.

$$
I = \int \sec(\theta) d\theta = \ln(\sec(\theta) + \tan(\theta)) + c
$$

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

Recall, tan $(\theta) = \frac{e^{x}}{2}$ $\frac{1}{3}$,

Example Evaluate $I = \int \frac{e^{x} dx}{\sqrt{2x}}$ $e^{2x} + 9$. Express you result in terms of the variable x.

Solution: So:
$$
I = \int \frac{\sec^2(\theta) d\theta}{|\sec(\theta)|}
$$
; $e^x = y = 3\tan(\theta)$; $\theta \in \left(0, \frac{\pi}{2}\right)$.

$$
I = \int \sec(\theta) d\theta = \ln\left(\sec(\theta) + \tan(\theta)\right) + c
$$

Recall,
$$
\tan(\theta) = \frac{e^x}{3}
$$
, hence $\sec(\theta) = \sqrt{\tan^2(\theta) + 1}$

Example Evaluate $I = \int \frac{e^{x} dx}{\sqrt{2x}}$ $e^{2x} + 9$. Express you result in terms of the variable x.

Solution: So:
$$
I = \int \frac{\sec^2(\theta) d\theta}{|\sec(\theta)|}
$$
; $e^x = y = 3 \tan(\theta)$; $\theta \in \left(0, \frac{\pi}{2}\right)$.

$$
I = \int \sec(\theta) \, d\theta = \ln\bigl(\sec(\theta) + \tan(\theta)\bigr) + c
$$

Recall, tan $(\theta) = \frac{e^{x}}{2}$ $\frac{e^{\lambda}}{3}$, hence sec $(\theta) = \sqrt{\tan^2(\theta) + 1} =$ $\int e^{2x}$ $\frac{1}{9} + 1.$

Example Evaluate $I = \int \frac{e^{x} dx}{\sqrt{2x}}$ $e^{2x} + 9$. Express you result in terms of the variable x.

Solution: So:
$$
I = \int \frac{\sec^2(\theta) d\theta}{|\sec(\theta)|}
$$
; $e^x = y = 3\tan(\theta)$; $\theta \in \left(0, \frac{\pi}{2}\right)$.

$$
I = \int \sec(\theta) \, d\theta = \ln(\sec(\theta) + \tan(\theta)) + c
$$

Recall, tan $(\theta) = \frac{e^{x}}{2}$ $\frac{e^{\lambda}}{3}$, hence sec $(\theta) = \sqrt{\tan^2(\theta) + 1} =$ $\int e^{2x}$ $\frac{1}{9} + 1.$

We conclude that.

$$
I = \ln\left(e^{x} + \frac{1}{3}\sqrt{e^{2x} + 9}\right) + c.
$$

K ロ X (日) X 제공 X 제공 X - 공 : X 이익(N)

Review for Exam 3.

- \blacktriangleright Trigonometric substitutions (8.3).
- Integration using partial fractions (8.4) .

KORK EX KEY KEY KORA

- \blacktriangleright L'Hôpital's rule (7.5).
- Improper integrals (8.7) .
- Infinite sequences (10.1) .

Recall: If the polynomial in the numerator has larger degree than the polynomial in the denominator, then do the long division first.

Recall: If the polynomial in the numerator has larger degree than the polynomial in the denominator, then do the long division first.

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

Example

Evaluate
$$
I = \int \frac{(x-2)}{(x^2 - x - 6)} dx.
$$

Recall: If the polynomial in the numerator has larger degree than the polynomial in the denominator, then do the long division first.

AD A REAKEN E VAN

Example

Evaluate
$$
I = \int \frac{(x-2)}{(x^2 - x - 6)} dx.
$$

Solution: We find the roots of the denominator,

Recall: If the polynomial in the numerator has larger degree than the polynomial in the denominator, then do the long division first.

Example

Evaluate
$$
I = \int \frac{(x-2)}{(x^2 - x - 6)} dx.
$$

Solution: We find the roots of the denominator, $x^2 - x - 6 = 0$,

Recall: If the polynomial in the numerator has larger degree than the polynomial in the denominator, then do the long division first.

Example

Evaluate
$$
I = \int \frac{(x-2)}{(x^2 - x - 6)} dx.
$$

Solution: We find the roots of the denominator, $x^2 - x - 6 = 0$,

$$
x_\pm=\frac{1}{2}\big(1\pm\sqrt{1+24}\big)
$$

Recall: If the polynomial in the numerator has larger degree than the polynomial in the denominator, then do the long division first.

Example

Evaluate
$$
I = \int \frac{(x-2)}{(x^2 - x - 6)} dx.
$$

Solution: We find the roots of the denominator, $x^2 - x - 6 = 0$,

$$
x_{\pm}=\frac{1}{2}\big(1\pm\sqrt{1+24}\big)=\frac{1}{2}(1\pm5)
$$

Recall: If the polynomial in the numerator has larger degree than the polynomial in the denominator, then do the long division first.

Example

Evaluate
$$
I = \int \frac{(x-2)}{(x^2 - x - 6)} dx.
$$

Solution: We find the roots of the denominator, $x^2 - x - 6 = 0$,

$$
x_{\pm} = \frac{1}{2} (1 \pm \sqrt{1+24}) = \frac{1}{2} (1 \pm 5) \Rightarrow \begin{cases} x_{+} = 3, \\ x_{-} = -2. \end{cases}
$$

AD A REAKEN E VAN

Recall: If the polynomial in the numerator has larger degree than the polynomial in the denominator, then do the long division first.

Example

Evaluate
$$
I = \int \frac{(x-2)}{(x^2-x-6)} dx.
$$

Solution: We find the roots of the denominator, $x^2 - x - 6 = 0$,

$$
x_{\pm} = \frac{1}{2} (1 \pm \sqrt{1+24}) = \frac{1}{2} (1 \pm 5) \Rightarrow \begin{cases} x_{+} = 3, \\ x_{-} = -2. \end{cases}
$$

AD A REAKEN E VAN

Therefore,
$$
I = \int \frac{(x-2)}{(x-3)(x+2)} dx.
$$

Recall: If the polynomial in the numerator has larger degree than the polynomial in the denominator, then do the long division first.

Example

Evaluate
$$
I = \int \frac{(x-2)}{(x^2-x-6)} dx.
$$

Solution: We find the roots of the denominator, $x^2 - x - 6 = 0$,

$$
x_{\pm} = \frac{1}{2} (1 \pm \sqrt{1+24}) = \frac{1}{2} (1 \pm 5) \Rightarrow \begin{cases} x_{+} = 3, \\ x_{-} = -2. \end{cases}
$$

ADAMPARATALE ARA

Therefore, $I = \int \dfrac{(x-2)}{(x-3)(x+2)} dx$. Now, partial fractions:

Recall: If the polynomial in the numerator has larger degree than the polynomial in the denominator, then do the long division first.

Example

Evaluate
$$
I = \int \frac{(x-2)}{(x^2 - x - 6)} dx.
$$

Solution: We find the roots of the denominator, $x^2 - x - 6 = 0$,

$$
x_{\pm} = \frac{1}{2} (1 \pm \sqrt{1+24}) = \frac{1}{2} (1 \pm 5) \Rightarrow \begin{cases} x_{+} = 3, \\ x_{-} = -2. \end{cases}
$$

Therefore, $I = \int \dfrac{(x-2)}{(x-3)(x+2)} dx$. Now, partial fractions: $(x - 2)$

 $(x - 3)(x + 2)$

Recall: If the polynomial in the numerator has larger degree than the polynomial in the denominator, then do the long division first.

Example

Evaluate
$$
I = \int \frac{(x-2)}{(x^2 - x - 6)} dx.
$$

Solution: We find the roots of the denominator, $x^2 - x - 6 = 0$,

$$
x_{\pm} = \frac{1}{2} (1 \pm \sqrt{1+24}) = \frac{1}{2} (1 \pm 5) \Rightarrow \begin{cases} x_{+} = 3, \\ x_{-} = -2. \end{cases}
$$

Therefore, $I = \int \dfrac{(x-2)}{(x-3)(x+2)} dx$. Now, partial fractions: $\frac{(x-2)}{(x-3)(x+2)} = \frac{a}{(x-3)} + \frac{b}{(x+1)}$ $(x + 2)$

Recall: If the polynomial in the numerator has larger degree than the polynomial in the denominator, then do the long division first.

Example

Evaluate
$$
I = \int \frac{(x-2)}{(x^2-x-6)} dx.
$$

Solution: We find the roots of the denominator, $x^2 - x - 6 = 0$,

$$
x_{\pm} = \frac{1}{2} (1 \pm \sqrt{1+24}) = \frac{1}{2} (1 \pm 5) \Rightarrow \begin{cases} x_{+} = 3, \\ x_{-} = -2. \end{cases}
$$

Therefore, $I = \int \dfrac{(x-2)}{(x-3)(x+2)} dx$. Now, partial fractions:

$$
\frac{(x-2)}{(x-3)(x+2)}=\frac{a}{(x-3)}+\frac{b}{(x+2)}\Rightarrow x-2=a(x+2)+b(x-3).
$$

Example

Evaluate
$$
I = \int \frac{(x-2)}{(x^2 - x - 6)} dx.
$$

Solution: Recall that:

$$
I = \int \left[\frac{a}{(x-3)} + \frac{b}{(x+2)} \right] dx; \ \ x - 2 = a(x+2) + b(x-3).
$$

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ | 할 | X 9 Q Q

Example

Evaluate
$$
I = \int \frac{(x-2)}{(x^2 - x - 6)} dx.
$$

Solution: Recall that:

$$
I = \int \left[\frac{a}{(x-3)} + \frac{b}{(x+2)} \right] dx; \ \ x - 2 = a(x+2) + b(x-3).
$$

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ 그럴 → 9 Q Q →

Evaluating at $x = 3$

Example

Evaluate
$$
I = \int \frac{(x-2)}{(x^2 - x - 6)} dx.
$$

Solution: Recall that:

$$
I = \int \left[\frac{a}{(x-3)} + \frac{b}{(x+2)} \right] dx; \ \ x - 2 = a(x+2) + b(x-3).
$$

Evaluating at $x = 3$ we get $a = \frac{1}{5}$,

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ | 할 | X 9 Q Q
Example

Evaluate
$$
I = \int \frac{(x-2)}{(x^2 - x - 6)} dx.
$$

Solution: Recall that:

$$
I = \int \left[\frac{a}{(x-3)} + \frac{b}{(x+2)} \right] dx; \ x - 2 = a(x+2) + b(x-3).
$$

Evaluating at $x = 3$ we get $a = \frac{1}{5}$, and at $x = -2$

Example

Evaluate
$$
I = \int \frac{(x-2)}{(x^2 - x - 6)} dx.
$$

Solution: Recall that:

$$
I = \int \left[\frac{a}{(x-3)} + \frac{b}{(x+2)} \right] dx; \ x - 2 = a(x+2) + b(x-3).
$$

Evaluating at $x = 3$ we get $a = \frac{1}{5}$, and at $x = -2$ we get $b = \frac{4}{5}$.

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 ⊙ Q Q ^

Example

Evaluate
$$
I = \int \frac{(x-2)}{(x^2 - x - 6)} dx.
$$

Solution: Recall that:

$$
I = \int \left[\frac{a}{(x-3)} + \frac{b}{(x+2)} \right] dx; \ \ x - 2 = a(x+2) + b(x-3).
$$

Evaluating at
$$
x = 3
$$
 we get $a = \frac{1}{5}$, and at $x = -2$ we get $b = \frac{4}{5}$.

$$
I=\frac{1}{5}\int\left[\frac{1}{(x-3)}+\frac{4}{(x+2)}\right]dx
$$

Example

Evaluate
$$
I = \int \frac{(x-2)}{(x^2 - x - 6)} dx.
$$

Solution: Recall that:

$$
I = \int \left[\frac{a}{(x-3)} + \frac{b}{(x+2)} \right] dx; \ \ x - 2 = a(x+2) + b(x-3).
$$

Evaluating at
$$
x = 3
$$
 we get $a = \frac{1}{5}$, and at $x = -2$ we get $b = \frac{4}{5}$.

$$
I = \frac{1}{5} \int \left[\frac{1}{(x-3)} + \frac{4}{(x+2)} \right] dx = \frac{1}{5} (\ln|x-3| + 4 \ln|x+2|) + c.
$$

Example

Evaluate
$$
I = \int \frac{(x-2)}{(x^2 - x - 6)} dx.
$$

Solution: Recall that:

$$
I = \int \left[\frac{a}{(x-3)} + \frac{b}{(x+2)} \right] dx; \ \ x - 2 = a(x+2) + b(x-3).
$$

Evaluating at
$$
x = 3
$$
 we get $a = \frac{1}{5}$, and at $x = -2$ we get $b = \frac{4}{5}$.

$$
I = \frac{1}{5} \int \left[\frac{1}{(x-3)} + \frac{4}{(x+2)} \right] dx = \frac{1}{5} (\ln|x-3| + 4 \ln|x+2|) + c.
$$

We conclude that $I = \ln(|x - 3|^{1/5}(x + 2)^{4/5}) + c$. △

Remark: Incomplete summary of partial fraction decompositions:

$$
\triangleright \frac{p_2(x)}{(x-r_1)(x-r_2)(x-r_3)} = \frac{c_1}{(x-r_1)} + \frac{c_2}{(x-r_2)} + \frac{c_3}{(x-r_3)}.
$$

Remark: Incomplete summary of partial fraction decompositions:

$$
\sum_{r} \frac{p_2(x)}{(x-r_1)(x-r_2)(x-r_3)} = \frac{c_1}{(x-r_1)} + \frac{c_2}{(x-r_2)} + \frac{c_3}{(x-r_3)}.
$$

$$
\sum_{r} \frac{p_2(x)}{(x-r_1)^3} = \frac{c_1}{(x-r_1)} + \frac{c_2}{(x-r_1)^2} + \frac{c_3}{(x-r_1)^3}.
$$

Remark: Incomplete summary of partial fraction decompositions:

$$
\sum_{r} \frac{p_2(x)}{(x-r_1)(x-r_2)(x-r_3)} = \frac{c_1}{(x-r_1)} + \frac{c_2}{(x-r_2)} + \frac{c_3}{(x-r_3)}.
$$
\n
$$
\sum_{r} \frac{p_2(x)}{(x-r_1)^3} = \frac{c_1}{(x-r_1)} + \frac{c_2}{(x-r_1)^2} + \frac{c_3}{(x-r_1)^3}.
$$
\n
$$
\sum_{r} \frac{p_2(x)}{(x-r_1)(x-r_2)^2} = \frac{c_1}{(x-r_1)} + \frac{c_2}{(x-r_2)} + \frac{(c_3x+c_4)}{(x-r_2)^2}.
$$

Remark: Incomplete summary of partial fraction decompositions:

$$
\sum_{r} \frac{p_2(x)}{(x-r_1)(x-r_2)(x-r_3)} = \frac{c_1}{(x-r_1)} + \frac{c_2}{(x-r_2)} + \frac{c_3}{(x-r_3)}.
$$
\n
$$
\sum_{r} \frac{p_2(x)}{(x-r_1)^3} = \frac{c_1}{(x-r_1)} + \frac{c_2}{(x-r_1)^2} + \frac{c_3}{(x-r_1)^3}.
$$
\n
$$
\sum_{r} \frac{p_2(x)}{(x-r_1)(x-r_2)^2} = \frac{c_1}{(x-r_1)} + \frac{c_2}{(x-r_2)} + \frac{(c_3x+c_4)}{(x-r_2)^2}.
$$
\n
$$
\sum_{r} \frac{p_5(x)}{(x^2+b^2)^3} = \frac{(c_1x+c_2)}{(x^2+b^2)} + \frac{(c_3x+c_4)}{(x^2+b^2)^2} + \frac{(c_5x+c_6)}{(x^2+b^2)^3}.
$$

Remark: Incomplete summary of partial fraction decompositions:

$$
\sum_{r} \frac{p_2(x)}{(x-r_1)(x-r_2)(x-r_3)} = \frac{c_1}{(x-r_1)} + \frac{c_2}{(x-r_2)} + \frac{c_3}{(x-r_3)}.
$$
\n
$$
\sum_{r} \frac{p_2(x)}{(x-r_1)^3} = \frac{c_1}{(x-r_1)} + \frac{c_2}{(x-r_1)^2} + \frac{c_3}{(x-r_1)^3}.
$$
\n
$$
\sum_{r} \frac{p_2(x)}{(x-r_1)(x-r_2)^2} = \frac{c_1}{(x-r_1)} + \frac{c_2}{(x-r_2)} + \frac{(c_3x+c_4)}{(x-r_2)^2}.
$$
\n
$$
\sum_{r} \frac{p_5(x)}{(x^2+b^2)^3} = \frac{(c_1x+c_2)}{(x^2+b^2)} + \frac{(c_3x+c_4)}{(x^2+b^2)^2} + \frac{(c_5x+c_6)}{(x^2+b^2)^3}.
$$
\n
$$
\sum_{r} \frac{p_4(x)}{(x-r_1)(x^2+b^2)^2} = \frac{c_1}{(x-r_1)} + \frac{(c_2x+c_3)}{(x^2+b^2)} + \frac{(c_4x+c_5)}{(x^2+b^2)^2}.
$$

Review for Exam 3.

- \blacktriangleright Trigonometric substitutions (8.3).
- Integration using partial fractions (8.4) .

KORK EX KEY KEY KORA

- \blacktriangleright L'Hôpital's rule (7.5).
- Improper integrals (8.7) .
- Infinite sequences (10.1) .

Example Evaluate the limit $L = \lim_{x \to \infty} \left(1 - \frac{2}{x} \right)$ $\frac{2}{x} - \frac{3}{x^2}$ $\frac{3}{x^2}\bigg)^{8x}$.

Example

Evaluate the limit
$$
L = \lim_{x \to \infty} \left(1 - \frac{2}{x} - \frac{3}{x^2}\right)^{8x}
$$
.

Solution: We first rewrite the limit as follows,

$$
L = \lim_{x \to \infty} \left(1 - \frac{2}{x} - \frac{3}{x^2} \right)^{8x}
$$

Example

Evaluate the limit
$$
L = \lim_{x \to \infty} \left(1 - \frac{2}{x} - \frac{3}{x^2}\right)^{8x}
$$
.

Solution: We first rewrite the limit as follows,

$$
L = \lim_{x \to \infty} \left(1 - \frac{2}{x} - \frac{3}{x^2} \right)^{8x} = \lim_{x \to \infty} e^{\left[8x \ln \left(1 - \frac{2}{x} - \frac{3}{x^2} \right) \right]}
$$

Example

Evaluate the limit
$$
L = \lim_{x \to \infty} \left(1 - \frac{2}{x} - \frac{3}{x^2}\right)^{8x}
$$
.

Solution: We first rewrite the limit as follows,

$$
L = \lim_{x \to \infty} \left(1 - \frac{2}{x} - \frac{3}{x^2} \right)^{8x} = \lim_{x \to \infty} e^{\left[8x \ln \left(1 - \frac{2}{x} - \frac{3}{x^2} \right) \right]}
$$

$$
L = e^{\lim_{x \to \infty} \left[8x \ln \left(1 - \frac{2}{x} - \frac{3}{x^2}\right)\right]}
$$

Example

Evaluate the limit
$$
L = \lim_{x \to \infty} \left(1 - \frac{2}{x} - \frac{3}{x^2}\right)^{8x}
$$
.

Solution: We first rewrite the limit as follows,

$$
L = \lim_{x \to \infty} \left(1 - \frac{2}{x} - \frac{3}{x^2} \right)^{8x} = \lim_{x \to \infty} e^{\left[8x \ln \left(1 - \frac{2}{x} - \frac{3}{x^2} \right) \right]}
$$

$$
L = e^{\lim_{x \to \infty} \left[8x \ln \left(1 - \frac{2}{x} - \frac{3}{x^2} \right) \right]} = e^{\lim_{x \to \infty} \left[\frac{\ln \left(1 - \frac{2}{x} - \frac{3}{x^2} \right)}{\frac{1}{8x}} \right]}
$$

Example

Evaluate the limit
$$
L = \lim_{x \to \infty} \left(1 - \frac{2}{x} - \frac{3}{x^2}\right)^{8x}
$$
.

Solution: We first rewrite the limit as follows,

$$
L = \lim_{x \to \infty} \left(1 - \frac{2}{x} - \frac{3}{x^2} \right)^{8x} = \lim_{x \to \infty} e^{\left[8x \ln \left(1 - \frac{2}{x} - \frac{3}{x^2} \right) \right]}
$$

$$
L = e^{\lim_{x \to \infty} \left[8x \ln \left(1 - \frac{2}{x} - \frac{3}{x^2} \right) \right]} = e^{\lim_{x \to \infty} \left[\frac{\ln \left(1 - \frac{2}{x} - \frac{3}{x^2} \right)}{\frac{1}{8x}} \right]}
$$

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ | 할 | X 9 Q Q

L'Hôpital rule in the exponent implies,

Example

Evaluate the limit
$$
L = \lim_{x \to \infty} \left(1 - \frac{2}{x} - \frac{3}{x^2}\right)^{8x}
$$
.

Solution: We first rewrite the limit as follows,

$$
L = \lim_{x \to \infty} \left(1 - \frac{2}{x} - \frac{3}{x^2} \right)^{8x} = \lim_{x \to \infty} e^{\left[8x \ln \left(1 - \frac{2}{x} - \frac{3}{x^2} \right) \right]}
$$

$$
L = e^{\lim_{x \to \infty} \left[8x \ln \left(1 - \frac{2}{x} - \frac{3}{x^2} \right) \right]} = e^{\lim_{x \to \infty} \left[\frac{\ln \left(1 - \frac{2}{x} - \frac{3}{x^2} \right)}{\frac{1}{8x}} \right]}
$$

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ | 할 | X 9 Q Q

L'Hôpital rule in the exponent implies,

$$
\lim_{x \to \infty} \frac{\ln \left(1 - \frac{2}{x} - \frac{3}{x^2}\right)}{\frac{1}{8x}}
$$

Example

Evaluate the limit
$$
L = \lim_{x \to \infty} \left(1 - \frac{2}{x} - \frac{3}{x^2}\right)^{8x}
$$
.

Solution: We first rewrite the limit as follows,

$$
L = \lim_{x \to \infty} \left(1 - \frac{2}{x} - \frac{3}{x^2} \right)^{8x} = \lim_{x \to \infty} e^{\left[8x \ln \left(1 - \frac{2}{x} - \frac{3}{x^2} \right) \right]}
$$

$$
L = e^{\lim_{x \to \infty} \left[8x \ln \left(1 - \frac{2}{x} - \frac{3}{x^2} \right) \right]} = e^{\lim_{x \to \infty} \left[\frac{\ln \left(1 - \frac{2}{x} - \frac{3}{x^2} \right)}{\frac{1}{8x}} \right]}
$$

L'Hôpital rule in the exponent implies,

$$
\lim_{x \to \infty} \frac{\ln \left(1 - \frac{2}{x} - \frac{3}{x^2} \right)}{\frac{1}{8x}} = \lim_{x \to \infty} \frac{\left(1 - \frac{2}{x} - \frac{3}{x^2} \right)^{-1} \left(\frac{2}{x^2} + \frac{6}{x^3} \right)}{\left(-\frac{1}{8x^2} \right)}
$$

Example Evaluate the limit $L = \lim_{x \to \infty} \left(1 - \frac{2}{x} \right)$ $\frac{2}{x} - \frac{3}{x^2}$ $\frac{3}{x^2}\bigg)^{8x}$.

Solution: Recall:
$$
L = e^{\lim_{x \to \infty} \left[\frac{\ln \left(1 - \frac{2}{x} - \frac{3}{x^2}\right)}{\frac{1}{8x}} \right]},
$$
 and

$$
\tilde{L} = \lim_{x \to \infty} \frac{\ln \left(1 - \frac{2}{x} - \frac{3}{x^2} \right)}{\frac{1}{8x}} = \lim_{x \to \infty} \frac{\left(1 - \frac{2}{x} - \frac{3}{x^2} \right)^{-1} \left(\frac{2}{x^2} + \frac{6}{x^3} \right)}{\left(-\frac{1}{8x^2} \right)}
$$

Example Evaluate the limit $L = \lim_{x \to \infty} \left(1 - \frac{2}{x} \right)$ $\frac{2}{x} - \frac{3}{x^2}$ $\frac{3}{x^2}\bigg)^{8x}$.

Solution: Recall:
$$
L = e^{\lim_{x \to \infty} \left[\frac{\ln \left(1 - \frac{2}{x} - \frac{3}{x^2}\right)}{\frac{1}{8x}} \right]},
$$
 and

$$
\tilde{L} = \lim_{x \to \infty} \frac{\ln \left(1 - \frac{2}{x} - \frac{3}{x^2} \right)}{\frac{1}{8x}} = \lim_{x \to \infty} \frac{\left(1 - \frac{2}{x} - \frac{3}{x^2} \right)^{-1} \left(\frac{2}{x^2} + \frac{6}{x^3} \right)}{\left(- \frac{1}{8x^2} \right)}
$$

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ (할) 1000

$$
\tilde{L} = \lim_{x \to \infty} -8\left(1 - \frac{2}{x} - \frac{3}{x^2}\right)^{-1} \left(2 + \frac{6}{x}\right)
$$

Example Evaluate the limit $L = \lim_{x \to \infty} \left(1 - \frac{2}{x} \right)$ $\frac{2}{x} - \frac{3}{x^2}$ $\frac{3}{x^2}\bigg)^{8x}$.

Solution: Recall:
$$
L = e^{\lim_{x \to \infty} \left[\frac{\ln \left(1 - \frac{2}{x} - \frac{3}{x^2}\right)}{\frac{1}{8x}} \right]},
$$
 and

$$
\tilde{L} = \lim_{x \to \infty} \frac{\ln \left(1 - \frac{2}{x} - \frac{3}{x^2} \right)}{\frac{1}{8x}} = \lim_{x \to \infty} \frac{\left(1 - \frac{2}{x} - \frac{3}{x^2} \right)^{-1} \left(\frac{2}{x^2} + \frac{6}{x^3} \right)}{\left(- \frac{1}{8x^2} \right)}
$$

$$
\tilde{L} = \lim_{x \to \infty} -8\left(1 - \frac{2}{x} - \frac{3}{x^2}\right)^{-1} \left(2 + \frac{6}{x}\right) = -16
$$

Example Evaluate the limit $L = \lim_{x \to \infty} \left(1 - \frac{2}{x} \right)$ $\frac{2}{x} - \frac{3}{x^2}$ $\frac{3}{x^2}\bigg)^{8x}$.

Solution: Recall:
$$
L = e^{\lim_{x \to \infty} \left[\frac{\ln \left(1 - \frac{2}{x} - \frac{3}{x^2}\right)}{\frac{1}{8x}} \right]},
$$
 and

$$
\tilde{L} = \lim_{x \to \infty} \frac{\ln \bigl(1 - \frac{2}{x} - \frac{3}{x^2} \bigr)}{\frac{1}{8x}} = \lim_{x \to \infty} \frac{\bigl(1 - \frac{2}{x} - \frac{3}{x^2} \bigr)^{-1} \bigl(\frac{2}{x^2} + \frac{6}{x^3} \bigr)}{\bigl(- \frac{1}{8x^2} \bigr)}
$$

$$
\tilde{L} = \lim_{x \to \infty} -8\left(1 - \frac{2}{x} - \frac{3}{x^2}\right)^{-1}\left(2 + \frac{6}{x}\right) = -16
$$

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ (할) 1000

We conclude that $L=e^{-16\theta}$. \triangleleft

Review for Exam 3.

- \blacktriangleright Trigonometric substitutions (8.3).
- Integration using partial fractions (8.4) .

KORK EX KEY KEY KORA

- \blacktriangleright L'Hôpital's rule (7.5).
- Improper integrals (8.7) .
- Infinite sequences (10.1) .

Example

Evaluate the integral
$$
I = \int_0^5 \frac{(x+1)}{\sqrt{25 - x^2}} dx
$$
.

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 | X 9 Q @

Example

Evaluate the integral
$$
I = \int_0^5 \frac{(x+1)}{\sqrt{25 - x^2}} dx
$$
.

Solution: We split the integral in two terms,

Example

Evaluate the integral
$$
I = \int_0^5 \frac{(x+1)}{\sqrt{25 - x^2}} dx
$$
.

Solution: We split the integral in two terms,

$$
I = \int_0^5 \frac{x}{\sqrt{25 - x^2}} dx + \int_0^5 \frac{1}{\sqrt{25 - x^2}} dx.
$$

Example

Evaluate the integral
$$
I = \int_0^5 \frac{(x+1)}{\sqrt{25 - x^2}} dx
$$
.

Solution: We split the integral in two terms,

$$
I = \int_0^5 \frac{x}{\sqrt{25 - x^2}} dx + \int_0^5 \frac{1}{\sqrt{25 - x^2}} dx.
$$

K ロ ▶ K @ ▶ K 할 X K 할 X 및 할 X 9 Q @

On the first term:

Example

Evaluate the integral
$$
I = \int_0^5 \frac{(x+1)}{\sqrt{25 - x^2}} dx
$$
.

Solution: We split the integral in two terms,

$$
I = \int_0^5 \frac{x}{\sqrt{25 - x^2}} dx + \int_0^5 \frac{1}{\sqrt{25 - x^2}} dx.
$$

K ロ ▶ K @ ▶ K 할 X K 할 X 및 할 X 9 Q @

On the first term: $u = 25 - x^2$,

Example

Evaluate the integral
$$
I = \int_0^5 \frac{(x+1)}{\sqrt{25 - x^2}} dx
$$
.

Solution: We split the integral in two terms,

$$
I = \int_0^5 \frac{x}{\sqrt{25 - x^2}} dx + \int_0^5 \frac{1}{\sqrt{25 - x^2}} dx.
$$

K ロ ▶ K @ ▶ K 할 X K 할 X 및 할 X 9 Q @

On the first term: $u = 25 - x^2$, $du = -2x dx$.

Example

Evaluate the integral
$$
I = \int_0^5 \frac{(x+1)}{\sqrt{25 - x^2}} dx
$$
.

Solution: We split the integral in two terms,

$$
I = \int_0^5 \frac{x}{\sqrt{25 - x^2}} dx + \int_0^5 \frac{1}{\sqrt{25 - x^2}} dx.
$$

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ (할) 수 있습니다

On the first term: $u = 25 - x^2$, $du = -2x dx$. Hence,

$$
I_1=\int_0^5\frac{x}{\sqrt{25-x^2}}\,dx
$$

Example

Evaluate the integral
$$
I = \int_0^5 \frac{(x+1)}{\sqrt{25 - x^2}} dx
$$
.

Solution: We split the integral in two terms,

$$
I = \int_0^5 \frac{x}{\sqrt{25 - x^2}} dx + \int_0^5 \frac{1}{\sqrt{25 - x^2}} dx.
$$

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ (할) 수 있습니다

On the first term: $u = 25 - x^2$, $du = -2x dx$. Hence,

$$
I_1 = \int_0^5 \frac{x}{\sqrt{25 - x^2}} dx = \int_{25}^0 -\frac{1}{\sqrt{u}} \frac{du}{2}
$$

Example

Evaluate the integral
$$
I = \int_0^5 \frac{(x+1)}{\sqrt{25 - x^2}} dx
$$
.

Solution: We split the integral in two terms,

$$
I = \int_0^5 \frac{x}{\sqrt{25 - x^2}} dx + \int_0^5 \frac{1}{\sqrt{25 - x^2}} dx.
$$

On the first term: $u = 25 - x^2$, $du = -2x dx$. Hence,

$$
I_1 = \int_0^5 \frac{x}{\sqrt{25 - x^2}} dx = \int_{25}^0 -\frac{1}{\sqrt{u}} \frac{du}{2} = \frac{1}{2} \int_0^{25} u^{-1/2} du.
$$

Example

Evaluate the integral
$$
I = \int_0^5 \frac{(x+1)}{\sqrt{25 - x^2}} dx
$$
.

Solution: We split the integral in two terms,

$$
I = \int_0^5 \frac{x}{\sqrt{25 - x^2}} dx + \int_0^5 \frac{1}{\sqrt{25 - x^2}} dx.
$$

On the first term: $u = 25 - x^2$, $du = -2x dx$. Hence,

$$
I_1 = \int_0^5 \frac{x}{\sqrt{25 - x^2}} dx = \int_{25}^0 -\frac{1}{\sqrt{u}} \frac{du}{2} = \frac{1}{2} \int_0^{25} u^{-1/2} du.
$$

$$
I_1 = \frac{1}{2} \lim_{c \to 0^+} \int_c^{25} u^{-1/2} du
$$

Example

Evaluate the integral
$$
I = \int_0^5 \frac{(x+1)}{\sqrt{25 - x^2}} dx
$$
.

Solution: We split the integral in two terms,

$$
I = \int_0^5 \frac{x}{\sqrt{25 - x^2}} dx + \int_0^5 \frac{1}{\sqrt{25 - x^2}} dx.
$$

On the first term: $u = 25 - x^2$, $du = -2x dx$. Hence,

$$
I_1 = \int_0^5 \frac{x}{\sqrt{25 - x^2}} dx = \int_{25}^0 -\frac{1}{\sqrt{u}} \frac{du}{2} = \frac{1}{2} \int_0^{25} u^{-1/2} du.
$$

$$
I_1 = \frac{1}{2} \lim_{c \to 0^+} \int_c^{25} u^{-1/2} du = \frac{1}{2} \lim_{c \to 0^+} 2u^{1/2} \Big|_c^{25}
$$

Example

Evaluate the integral
$$
I = \int_0^5 \frac{(x+1)}{\sqrt{25 - x^2}} dx
$$
.

Solution: We split the integral in two terms,

$$
I = \int_0^5 \frac{x}{\sqrt{25 - x^2}} dx + \int_0^5 \frac{1}{\sqrt{25 - x^2}} dx.
$$

On the first term: $u = 25 - x^2$, $du = -2x dx$. Hence,

$$
I_1 = \int_0^5 \frac{x}{\sqrt{25 - x^2}} dx = \int_{25}^0 -\frac{1}{\sqrt{u}} \frac{du}{2} = \frac{1}{2} \int_0^{25} u^{-1/2} du.
$$

$$
I_1 = \frac{1}{2} \lim_{c \to 0^+} \int_c^{25} u^{-1/2} du = \frac{1}{2} \lim_{c \to 0^+} 2u^{1/2} \Big|_c^{25} \quad \Rightarrow \quad I_1 = 5.
$$
Example

Evaluate the integral
$$
I = \int_0^5 \frac{(x+1)}{\sqrt{25 - x^2}} dx
$$
.

Solution: Recall:
$$
I_1 = \int_0^5 \frac{x}{\sqrt{25 - x^2}} dx = 5.
$$

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 | X 9 Q @

Example

Evaluate the integral
$$
I = \int_0^5 \frac{(x+1)}{\sqrt{25 - x^2}} dx
$$
.

Solution: Recall:
$$
I_1 = \int_0^5 \frac{x}{\sqrt{25 - x^2}} dx = 5.
$$

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ | 할 | X 9 Q Q

In the second integral:

Example

Evaluate the integral
$$
I = \int_0^5 \frac{(x+1)}{\sqrt{25 - x^2}} dx
$$
.

Solution: Recall: $I_1 = \int_0^5$ 0 $\frac{x}{\sqrt{2-x^2}}$ $\frac{1}{25-x^2} dx = 5.$

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

In the second integral: $x = 5 \sin(\theta)$,

Example

Evaluate the integral
$$
I = \int_0^5 \frac{(x+1)}{\sqrt{25 - x^2}} dx
$$
.

Solution: Recall: $I_1 = \int_0^5$ 0 $\frac{x}{\sqrt{2-x^2}}$ $\frac{1}{25-x^2} dx = 5.$

In the second integral: $x = 5 \sin(\theta)$, $dx = 5 \cos(\theta) d\theta$;

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ (할) 1000

Example

Evaluate the integral
$$
I = \int_0^5 \frac{(x+1)}{\sqrt{25 - x^2}} dx
$$
.

Solution: Recall: $I_1 = \int_0^5$ 0 $\frac{x}{\sqrt{2-x^2}}$ $\frac{1}{25-x^2} dx = 5.$

In the second integral: $x = 5 \sin(\theta)$, $dx = 5 \cos(\theta) d\theta$; Hence

K □ ▶ K @ ▶ K 할 X K 할 X (할) 10 Q Q Q

$$
I_2=\int_0^5\frac{dx}{\sqrt{25-x^2}}
$$

Example

Evaluate the integral
$$
I = \int_0^5 \frac{(x+1)}{\sqrt{25 - x^2}} dx
$$
.

Solution: Recall: $I_1 = \int_0^5$ 0 $\frac{x}{\sqrt{2-x^2}}$ $\frac{1}{25-x^2} dx = 5.$

In the second integral: $x = 5 \sin(\theta)$, $dx = 5 \cos(\theta) d\theta$; Hence

$$
I_2 = \int_0^5 \frac{dx}{\sqrt{25 - x^2}} = \int_0^{\pi/2} \frac{5 \cos(\theta) d\theta}{\sqrt{25 - 25 \sin^2(\theta)}}
$$

K □ ▶ K @ ▶ K 할 X K 할 X (할) 10 Q Q Q

Example

Evaluate the integral
$$
I = \int_0^5 \frac{(x+1)}{\sqrt{25 - x^2}} dx
$$
.

Solution: Recall: $I_1 = \int_0^5$ 0 $\frac{x}{\sqrt{2-x^2}}$ $\frac{1}{25-x^2} dx = 5.$

In the second integral: $x = 5 \sin(\theta)$, $dx = 5 \cos(\theta) d\theta$; Hence

$$
I_2 = \int_0^5 \frac{dx}{\sqrt{25 - x^2}} = \int_0^{\pi/2} \frac{5 \cos(\theta) d\theta}{\sqrt{25 - 25 \sin^2(\theta)}}
$$

KD ▶ K@ ▶ K 통 K K 통 K - ⊙ Q Q ^

$$
I_2 = \int_0^{\pi/2} \frac{\cos(\theta)}{|\cos(\theta)|} d\theta
$$

Example

Evaluate the integral
$$
I = \int_0^5 \frac{(x+1)}{\sqrt{25 - x^2}} dx
$$
.

Solution: Recall: $I_1 = \int_0^5$ 0 $\frac{x}{\sqrt{2-x^2}}$ $\frac{1}{25-x^2} dx = 5.$

In the second integral: $x = 5 \sin(\theta)$, $dx = 5 \cos(\theta) d\theta$; Hence

$$
I_2 = \int_0^5 \frac{dx}{\sqrt{25 - x^2}} = \int_0^{\pi/2} \frac{5 \cos(\theta) d\theta}{\sqrt{25 - 25 \sin^2(\theta)}}
$$

K □ ▶ K @ ▶ K 할 X K 할 X (할) 10 Q Q Q

$$
I_2 = \int_0^{\pi/2} \frac{\cos(\theta)}{|\cos(\theta)|} \, d\theta = \int_0^{\pi/2} d\theta
$$

Example

Evaluate the integral
$$
I = \int_0^5 \frac{(x+1)}{\sqrt{25 - x^2}} dx
$$
.

Solution: Recall: $I_1 = \int_0^5$ 0 $\frac{x}{\sqrt{2-x^2}}$ $\frac{1}{25-x^2} dx = 5.$

In the second integral: $x = 5 \sin(\theta)$, $dx = 5 \cos(\theta) d\theta$; Hence

$$
I_2 = \int_0^5 \frac{dx}{\sqrt{25 - x^2}} = \int_0^{\pi/2} \frac{5 \cos(\theta) d\theta}{\sqrt{25 - 25 \sin^2(\theta)}}
$$

$$
I_2 = \int_0^{\pi/2} \frac{\cos(\theta)}{|\cos(\theta)|} d\theta = \int_0^{\pi/2} d\theta \quad \Rightarrow \quad I_2 = \frac{\pi}{2}.
$$

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ (할) 1000

Example

Evaluate the integral
$$
I = \int_0^5 \frac{(x+1)}{\sqrt{25 - x^2}} dx
$$
.

Solution: Recall: $I_1 = \int_0^5$ 0 $\frac{x}{\sqrt{2-x^2}}$ $\frac{1}{25-x^2} dx = 5.$

In the second integral: $x = 5 \sin(\theta)$, $dx = 5 \cos(\theta) d\theta$; Hence

$$
I_2 = \int_0^5 \frac{dx}{\sqrt{25 - x^2}} = \int_0^{\pi/2} \frac{5 \cos(\theta) d\theta}{\sqrt{25 - 25 \sin^2(\theta)}}
$$

$$
I_2 = \int_0^{\pi/2} \frac{\cos(\theta)}{|\cos(\theta)|} d\theta = \int_0^{\pi/2} d\theta \quad \Rightarrow \quad I_2 = \frac{\pi}{2}.
$$

AD A REAKEN E VAN

We conclude that $I = 5 + \frac{\pi}{2}$ 2 \sim C \sim C \sim C \sim C \sim C \sim

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ | 할 | X 9 Q Q

 \blacktriangleright Direct comparison test:

 \blacktriangleright Direct comparison test:

If $0 \leq f(x) \leq g(x)$ for $x \in [a,\infty)$, then holds

$$
0\leqslant \int_a^\infty f(x)\,dx\leqslant \int_a^\infty g(x)\,dx.
$$

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

 \blacktriangleright Direct comparison test:

If $0 \leq f(x) \leq g(x)$ for $x \in [a,\infty)$, then holds $0\leqslant\int^\infty$ a $f(x) dx \leqslant \int_{0}^{\infty}$ a $g(x)$ dx. (a) \int^{∞} a $g(x)$ dx converges \Rightarrow $\int_{-\infty}^{\infty}$ a $f(x)$ dx converges; (b) \int^{∞} a $f(x)$ dx diverges \Rightarrow \int_{0}^{∞} a $g(x)$ dx diverges.

4 D > 4 P > 4 E > 4 E > E + 9 Q O

 \blacktriangleright Direct comparison test:

If $0 \le f(x) \le g(x)$ for $x \in [a,\infty)$, then holds $0\leqslant\int^\infty$ a $f(x) dx \leqslant \int_{0}^{\infty}$ a $g(x)$ dx. (a) \int^{∞} a $g(x)$ dx converges \Rightarrow $\int_{-\infty}^{\infty}$ a $f(x)$ dx converges; (b) \int^{∞} a $f(x)$ dx diverges \Rightarrow \int_{0}^{∞} a $g(x)$ dx diverges.

4 D > 4 P > 4 E > 4 E > E + 9 Q O

 \blacktriangleright Limit comparison test:

 \blacktriangleright Direct comparison test:

If $0 \le f(x) \le g(x)$ for $x \in [a,\infty)$, then holds $0\leqslant\int^\infty$ a $f(x) dx \leqslant \int_{0}^{\infty}$ a $g(x)$ dx. (a) \int^{∞} a $g(x)$ dx converges \Rightarrow $\int_{-\infty}^{\infty}$ a $f(x)$ dx converges; (b) \int^{∞} a $f(x)$ dx diverges \Rightarrow \int_{0}^{∞} a $g(x)$ dx diverges.

4 D > 4 P > 4 E > 4 E > E + 9 Q O

 \blacktriangleright Limit comparison test:

If
$$
\lim_{x \to \infty} \frac{f(x)}{g(x)} = L
$$
, with $0 < L < \infty$,

 \blacktriangleright Direct comparison test:

If $0 \leq f(x) \leq g(x)$ for $x \in [a,\infty)$, then holds $0\leqslant\int^\infty$ a $f(x) dx \leqslant \int_{0}^{\infty}$ a $g(x)$ dx. (a) \int^{∞} a $g(x)$ dx converges \Rightarrow $\int_{-\infty}^{\infty}$ a $f(x)$ dx converges; (b) \int^{∞} a $f(x)$ dx diverges \Rightarrow \int_{0}^{∞} a $g(x)$ dx diverges.

 \blacktriangleright Limit comparison test:

If
$$
\lim_{x \to \infty} \frac{f(x)}{g(x)} = L
$$
, with $0 < L < \infty$, then the integrals

$$
\int_{a}^{\infty} f(x) dx, \qquad \int_{a}^{\infty} g(x) dx
$$

4 D > 4 P > 4 E > 4 E > E + 9 Q O

both converge or both diverge.

Example Determine whether $I = \int_{0}^{\infty}$ 3 $\frac{x dx}{\sqrt{2}}$ $\frac{x}{x^5 + x^3}$ converges or not.

Example

Determine whether
$$
I = \int_3^\infty \frac{x \, dx}{\sqrt{x^5 + x^3}}
$$
 converges or not.

Solution: First, find an appropriate function $g(x)$ such that:

K ロ ▶ K @ ▶ K 할 X K 할 X 및 할 X 9 Q @

Example

Determine whether $I = \int_{0}^{\infty}$ 3 $\frac{x dx}{\sqrt{2}}$ $\frac{x}{x^5 + x^3}$ converges or not.

Solution: First, find an appropriate function $g(x)$ such that:

$$
\lim_{x \to \infty} g(x) = \lim_{x \to \infty} \frac{x}{\sqrt{x^5 + x^3}}
$$

Example

Determine whether $I = \int_{0}^{\infty}$ 3 $\frac{x dx}{\sqrt{2}}$ $\frac{x}{x^5 + x^3}$ converges or not.

Solution: First, find an appropriate function $g(x)$ such that:

$$
\lim_{x \to \infty} g(x) = \lim_{x \to \infty} \frac{x}{\sqrt{x^5 + x^3}} = \lim_{x \to \infty} \frac{x}{x^{5/2}}
$$

Example

Determine whether $I = \int_{0}^{\infty}$ 3 $\frac{x dx}{\sqrt{2}}$ $\frac{x}{x^5 + x^3}$ converges or not.

Solution: First, find an appropriate function $g(x)$ such that:

$$
\lim_{x \to \infty} g(x) = \lim_{x \to \infty} \frac{x}{\sqrt{x^5 + x^3}} = \lim_{x \to \infty} \frac{x}{x^{5/2}} = \lim_{x \to \infty} \frac{1}{x^{3/2}}.
$$

Example

Determine whether $I = \int_{0}^{\infty}$ 3 $\frac{x dx}{\sqrt{2}}$ $\frac{x}{x^5 + x^3}$ converges or not.

Solution: First, find an appropriate function $g(x)$ such that:

$$
\lim_{x \to \infty} g(x) = \lim_{x \to \infty} \frac{x}{\sqrt{x^5 + x^3}} = \lim_{x \to \infty} \frac{x}{x^{5/2}} = \lim_{x \to \infty} \frac{1}{x^{3/2}}.
$$

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

Therefore, we use the limit comparison test with $g(x) = x^{-3/2}$.

Example

Determine whether $I = \int_{0}^{\infty}$ 3 $\frac{x dx}{\sqrt{2}}$ $\frac{x}{x^5 + x^3}$ converges or not.

Solution: First, find an appropriate function $g(x)$ such that:

$$
\lim_{x \to \infty} g(x) = \lim_{x \to \infty} \frac{x}{\sqrt{x^5 + x^3}} = \lim_{x \to \infty} \frac{x}{x^{5/2}} = \lim_{x \to \infty} \frac{1}{x^{3/2}}.
$$

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

Therefore, we use the limit comparison test with $g(x) = x^{-3/2}$. Then, by construction,

Example

Determine whether $I = \int_{0}^{\infty}$ 3 $\frac{x dx}{\sqrt{2}}$ $\frac{x}{x^5 + x^3}$ converges or not.

Solution: First, find an appropriate function $g(x)$ such that:

$$
\lim_{x \to \infty} g(x) = \lim_{x \to \infty} \frac{x}{\sqrt{x^5 + x^3}} = \lim_{x \to \infty} \frac{x}{x^{5/2}} = \lim_{x \to \infty} \frac{1}{x^{3/2}}.
$$

K ロ X (日) X 제공 X 제공 X - 공 : X 이익(N)

Therefore, we use the limit comparison test with $g(x) = x^{-3/2}$. Then, by construction,

$$
\lim_{x \to \infty} \left(\frac{x}{\sqrt{x^5 + x^3}} \right) \left(\frac{1}{x^{-3/2}} \right)
$$

Example

Determine whether $I = \int_{0}^{\infty}$ 3 $\frac{x dx}{\sqrt{2}}$ $\frac{x}{x^5 + x^3}$ converges or not.

Solution: First, find an appropriate function $g(x)$ such that:

$$
\lim_{x \to \infty} g(x) = \lim_{x \to \infty} \frac{x}{\sqrt{x^5 + x^3}} = \lim_{x \to \infty} \frac{x}{x^{5/2}} = \lim_{x \to \infty} \frac{1}{x^{3/2}}.
$$

Therefore, we use the limit comparison test with $g(x) = x^{-3/2}$. Then, by construction,

$$
\lim_{x \to \infty} \left(\frac{x}{\sqrt{x^5 + x^3}} \right) \left(\frac{1}{x^{-3/2}} \right) = \lim_{x \to \infty} \left(\frac{x}{x^{5/2}} \right) \left(\frac{1}{x^{-3/2}} \right)
$$

Example

Determine whether $I = \int_{0}^{\infty}$ 3 $\frac{x dx}{\sqrt{2}}$ $\frac{x}{x^5 + x^3}$ converges or not.

Solution: First, find an appropriate function $g(x)$ such that:

$$
\lim_{x \to \infty} g(x) = \lim_{x \to \infty} \frac{x}{\sqrt{x^5 + x^3}} = \lim_{x \to \infty} \frac{x}{x^{5/2}} = \lim_{x \to \infty} \frac{1}{x^{3/2}}.
$$

Therefore, we use the limit comparison test with $g(x) = x^{-3/2}$. Then, by construction,

$$
\lim_{x \to \infty} \Big(\frac{x}{\sqrt{x^5 + x^3}} \Big) \Big(\frac{1}{x^{-3/2}} \Big) = \lim_{x \to \infty} \Big(\frac{x}{x^{5/2}} \Big) \Big(\frac{1}{x^{-3/2}} \Big) = 1.
$$

Example

Determine whether $I = \int_{0}^{\infty}$ 3 $\frac{x dx}{\sqrt{2}}$ $\frac{x}{x^5 + x^3}$ converges or not.

Solution: First, find an appropriate function $g(x)$ such that:

$$
\lim_{x \to \infty} g(x) = \lim_{x \to \infty} \frac{x}{\sqrt{x^5 + x^3}} = \lim_{x \to \infty} \frac{x}{x^{5/2}} = \lim_{x \to \infty} \frac{1}{x^{3/2}}.
$$

Therefore, we use the limit comparison test with $g(x) = x^{-3/2}$. Then, by construction,

$$
\lim_{x \to \infty} \left(\frac{x}{\sqrt{x^5 + x^3}} \right) \left(\frac{1}{x^{-3/2}} \right) = \lim_{x \to \infty} \left(\frac{x}{x^{5/2}} \right) \left(\frac{1}{x^{-3/2}} \right) = 1.
$$

K ロ X (日) X 제공 X 제공 X - 공 : X 이익(N)

Since
$$
\int_3^{\infty} x^{-3/2} dx = -2x^{-1/2}\Big|_3^{\infty}
$$

Example

Determine whether $I = \int_{0}^{\infty}$ 3 $\frac{x dx}{\sqrt{2}}$ $\frac{x}{x^5 + x^3}$ converges or not.

Solution: First, find an appropriate function $g(x)$ such that:

$$
\lim_{x \to \infty} g(x) = \lim_{x \to \infty} \frac{x}{\sqrt{x^5 + x^3}} = \lim_{x \to \infty} \frac{x}{x^{5/2}} = \lim_{x \to \infty} \frac{1}{x^{3/2}}.
$$

Therefore, we use the limit comparison test with $g(x) = x^{-3/2}$. Then, by construction,

$$
\lim_{x \to \infty} \left(\frac{x}{\sqrt{x^5 + x^3}} \right) \left(\frac{1}{x^{-3/2}} \right) = \lim_{x \to \infty} \left(\frac{x}{x^{5/2}} \right) \left(\frac{1}{x^{-3/2}} \right) = 1.
$$

K ロ X (日) X 제공 X 제공 X - 공 : X 이익(N)

Since
$$
\int_3^{\infty} x^{-3/2} dx = -2x^{-1/2} \Big|_3^{\infty} = -2\left(0 - \frac{1}{\sqrt{3}}\right)
$$

Example

Determine whether $I = \int_{0}^{\infty}$ 3 $\frac{x dx}{\sqrt{2}}$ $\frac{x}{x^5 + x^3}$ converges or not.

Solution: First, find an appropriate function $g(x)$ such that:

$$
\lim_{x \to \infty} g(x) = \lim_{x \to \infty} \frac{x}{\sqrt{x^5 + x^3}} = \lim_{x \to \infty} \frac{x}{x^{5/2}} = \lim_{x \to \infty} \frac{1}{x^{3/2}}.
$$

Therefore, we use the limit comparison test with $g(x) = x^{-3/2}$. Then, by construction,

$$
\lim_{x \to \infty} \left(\frac{x}{\sqrt{x^5 + x^3}} \right) \left(\frac{1}{x^{-3/2}} \right) = \lim_{x \to \infty} \left(\frac{x}{x^{5/2}} \right) \left(\frac{1}{x^{-3/2}} \right) = 1.
$$

Since
$$
\int_3^\infty x^{-3/2} dx = -2x^{-1/2}\Big|_3^\infty = -2\Big(0 - \frac{1}{\sqrt{3}}\Big) = \frac{2}{\sqrt{3}}
$$

Example

Determine whether $I = \int_{0}^{\infty}$ 3 $\frac{x dx}{\sqrt{2}}$ $\frac{x}{x^5 + x^3}$ converges or not.

Solution: First, find an appropriate function $g(x)$ such that:

$$
\lim_{x \to \infty} g(x) = \lim_{x \to \infty} \frac{x}{\sqrt{x^5 + x^3}} = \lim_{x \to \infty} \frac{x}{x^{5/2}} = \lim_{x \to \infty} \frac{1}{x^{3/2}}.
$$

Therefore, we use the limit comparison test with $g(x) = x^{-3/2}$. Then, by construction,

$$
\lim_{x \to \infty} \left(\frac{x}{\sqrt{x^5 + x^3}} \right) \left(\frac{1}{x^{-3/2}} \right) = \lim_{x \to \infty} \left(\frac{x}{x^{5/2}} \right) \left(\frac{1}{x^{-3/2}} \right) = 1.
$$

Since
$$
\int_3^{\infty} x^{-3/2} dx = -2x^{-1/2}\Big|_3^{\infty} = -2\Big(0 - \frac{1}{\sqrt{3}}\Big) = \frac{2}{\sqrt{3}}
$$
,

we conclude that ℓ converges.

Example

Determine whether $I = \int_{0}^{\infty}$ 3 $\frac{x dx}{\sqrt{2}}$ $\frac{x}{x^5 + x^3}$ converges or not.

Example

Determine whether $I = \int_{0}^{\infty}$ 3 $\frac{x dx}{\sqrt{2}}$ $\frac{x}{x^5 + x^3}$ converges or not.

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

Solution: We use the Direct Comparison Test:

Example

Determine whether
$$
I = \int_3^{\infty} \frac{x \, dx}{\sqrt{x^5 + x^3}}
$$
 converges or not.

Solution: We use the Direct Comparison Test: For $x > 0$ holds

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ (할) 수 있습니다

Example

Determine whether
$$
I = \int_3^{\infty} \frac{x \, dx}{\sqrt{x^5 + x^3}}
$$
 converges or not.

Solution: We use the Direct Comparison Test: For $x > 0$ holds

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ (할) 수 있습니다

$$
x^5 < x^5 + x^3
$$

Example

Determine whether
$$
I = \int_3^{\infty} \frac{x \, dx}{\sqrt{x^5 + x^3}}
$$
 converges or not.

Solution: We use the Direct Comparison Test: For $x > 0$ holds

$$
x^5 < x^5 + x^3 \quad \Rightarrow \quad \frac{1}{x^5 + x^3} < \frac{1}{x^5}
$$

K ロ ▶ K @ ▶ K 할 X K 할 X 및 할 X 9 Q @

Example

Determine whether
$$
I = \int_3^\infty \frac{x \, dx}{\sqrt{x^5 + x^3}}
$$
 converges or not.

Solution: We use the Direct Comparison Test: For $x > 0$ holds

$$
x^5 < x^5 + x^3 \quad \Rightarrow \quad \frac{1}{x^5 + x^3} < \frac{1}{x^5}
$$

$$
\frac{1}{\sqrt{x^5+x^3}}<\frac{1}{\sqrt{x^5}}
$$
Example

Determine whether
$$
I = \int_3^\infty \frac{x \, dx}{\sqrt{x^5 + x^3}}
$$
 converges or not.

Solution: We use the Direct Comparison Test: For $x > 0$ holds

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

$$
x^5 < x^5 + x^3 \quad \Rightarrow \quad \frac{1}{x^5 + x^3} < \frac{1}{x^5}
$$
\n
$$
\frac{1}{\sqrt{x^5 + x^3}} < \frac{1}{\sqrt{x^5}} = x^{-5/2}
$$

Example

Determine whether
$$
I = \int_3^\infty \frac{x \, dx}{\sqrt{x^5 + x^3}}
$$
 converges or not.

Solution: We use the Direct Comparison Test: For $x > 0$ holds

$$
x^5 < x^5 + x^3 \quad \Rightarrow \quad \frac{1}{x^5 + x^3} < \frac{1}{x^5}
$$
\n
$$
\frac{1}{\sqrt{x^5 + x^3}} < \frac{1}{\sqrt{x^5}} = x^{-5/2} \Rightarrow \frac{x}{\sqrt{x^5 + x^3}} < x^{-5/2}x
$$

Example

Determine whether
$$
I = \int_3^\infty \frac{x \, dx}{\sqrt{x^5 + x^3}}
$$
 converges or not.

Solution: We use the Direct Comparison Test: For $x > 0$ holds

$$
x^5 < x^5 + x^3 \quad \Rightarrow \quad \frac{1}{x^5 + x^3} < \frac{1}{x^5}
$$
\n
$$
\frac{1}{\sqrt{x^5 + x^3}} < \frac{1}{\sqrt{x^5}} = x^{-5/2} \Rightarrow \frac{x}{\sqrt{x^5 + x^3}} < x^{-5/2}x = x^{-3/2}.
$$

Example

Determine whether
$$
I = \int_3^\infty \frac{x \, dx}{\sqrt{x^5 + x^3}}
$$
 converges or not.

Solution: We use the Direct Comparison Test: For $x > 0$ holds

$$
x^{5} < x^{5} + x^{3} \Rightarrow \frac{1}{x^{5} + x^{3}} < \frac{1}{x^{5}}
$$

$$
\frac{1}{\sqrt{x^{5} + x^{3}}} < \frac{1}{\sqrt{x^{5}}} = x^{-5/2} \Rightarrow \frac{x}{\sqrt{x^{5} + x^{3}}} < x^{-5/2}x = x^{-3/2}.
$$

$$
1 < \int_{3}^{\infty} x^{-3/2} dx
$$

K ロ ▶ K @ ▶ K 할 X K 할 X 및 할 X 9 Q @

Example

Determine whether
$$
I = \int_3^\infty \frac{x \, dx}{\sqrt{x^5 + x^3}}
$$
 converges or not.

Solution: We use the Direct Comparison Test: For $x > 0$ holds

$$
x^{5} < x^{5} + x^{3} \Rightarrow \frac{1}{x^{5} + x^{3}} < \frac{1}{x^{5}}
$$

$$
\frac{1}{\sqrt{x^{5} + x^{3}}} < \frac{1}{\sqrt{x^{5}}} = x^{-5/2} \Rightarrow \frac{x}{\sqrt{x^{5} + x^{3}}} < x^{-5/2}x = x^{-3/2}.
$$

$$
l < \int_3^\infty x^{-3/2} \, dx = -2 \, x^{-1/2} \Big|_3^\infty
$$

Example

Determine whether
$$
I = \int_3^\infty \frac{x \, dx}{\sqrt{x^5 + x^3}}
$$
 converges or not.

Solution: We use the Direct Comparison Test: For $x > 0$ holds

$$
x^{5} < x^{5} + x^{3} \Rightarrow \frac{1}{x^{5} + x^{3}} < \frac{1}{x^{5}}
$$

$$
\frac{1}{\sqrt{x^{5} + x^{3}}} < \frac{1}{\sqrt{x^{5}}} = x^{-5/2} \Rightarrow \frac{x}{\sqrt{x^{5} + x^{3}}} < x^{-5/2}x = x^{-3/2}.
$$

$$
1 < \int_3^\infty x^{-3/2} \, dx = -2x^{-1/2} \Big|_3^\infty = -2\Big(0 - \frac{1}{\sqrt{3}}\Big)
$$

Example

Determine whether
$$
I = \int_3^\infty \frac{x \, dx}{\sqrt{x^5 + x^3}}
$$
 converges or not.

Solution: We use the Direct Comparison Test: For $x > 0$ holds

$$
x^5 < x^5 + x^3 \quad \Rightarrow \quad \frac{1}{x^5 + x^3} < \frac{1}{x^5}
$$
\n
$$
\frac{1}{\sqrt{x^5 + x^3}} < \frac{1}{\sqrt{x^5}} = x^{-5/2} \Rightarrow \frac{x}{\sqrt{x^5 + x^3}} < x^{-5/2}x = x^{-3/2}.
$$

$$
1 < \int_3^{\infty} x^{-3/2} dx = -2x^{-1/2} \Big|_3^{\infty} = -2\Big(0 - \frac{1}{\sqrt{3}}\Big) = \frac{2}{\sqrt{3}}.
$$

Example

Determine whether
$$
I = \int_3^\infty \frac{x \, dx}{\sqrt{x^5 + x^3}}
$$
 converges or not.

Solution: We use the Direct Comparison Test: For $x > 0$ holds

$$
x^5 < x^5 + x^3 \quad \Rightarrow \quad \frac{1}{x^5 + x^3} < \frac{1}{x^5}
$$
\n
$$
\frac{1}{\sqrt{x^5 + x^3}} < \frac{1}{\sqrt{x^5}} = x^{-5/2} \Rightarrow \frac{x}{\sqrt{x^5 + x^3}} < x^{-5/2}x = x^{-3/2}.
$$
\n
$$
1 < \int_3^\infty x^{-3/2} \, dx = -2x^{-1/2} \Big|_3^\infty = -2\left(0 - \frac{1}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}}.
$$

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ (할) 1000

We conclude that I converges. \Box

Review for Exam 3.

- \blacktriangleright Trigonometric substitutions (8.3).
- Integration using partial fractions (8.4) .

KORK EX KEY KEY KORA

- \blacktriangleright L'Hôpital's rule (7.5).
- Improper integrals (8.7) .
- Infinite sequences (10.1) .

Example Evaluate $L = \lim_{n \to \infty} \left(\frac{8}{3n} \right)$ $\bigg\}^{\frac{1}{3n}}$.

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ | 할 | X 9 Q Q

Example

Evaluate
$$
L = \lim_{n \to \infty} \left(\frac{8}{3n}\right)^{\frac{1}{3n}}
$$
.

Solution: We study a similar limit for the function $f(x) = \left(\frac{8}{3}\right)^2$ $\frac{8}{3x}$ $\frac{1}{3x}$.

K ロ ▶ K @ ▶ K 할 X K 할 X 및 할 X 9 Q @

Example

Evaluate
$$
L = \lim_{n \to \infty} \left(\frac{8}{3n}\right)^{\frac{1}{3n}}
$$
.

Solution: We study a similar limit for the function $f(x) = \left(\frac{8}{3}\right)^2$ $\frac{8}{3x}$ $\frac{1}{3x}$.

K □ ▶ K @ ▶ K 할 X K 할 X (할) 10 Q Q Q

$$
\lim_{x\to\infty}\left(\frac{8}{3x}\right)^{\frac{1}{3x}}
$$

Example

Evaluate
$$
L = \lim_{n \to \infty} \left(\frac{8}{3n}\right)^{\frac{1}{3n}}
$$
.

Solution: We study a similar limit for the function $f(x) = \left(\frac{8}{3}\right)^2$ $\frac{8}{3x}$ $\frac{1}{3x}$.

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ | 할 | X 9 Q Q

$$
\lim_{x \to \infty} \left(\frac{8}{3x}\right)^{\frac{1}{3x}} = \lim_{x \to \infty} e^{\left[\frac{\ln\left(\frac{8}{3x}\right)}{3x}\right]}
$$

Example

Evaluate
$$
L = \lim_{n \to \infty} \left(\frac{8}{3n}\right)^{\frac{1}{3n}}
$$
.

Solution: We study a similar limit for the function $f(x) = \left(\frac{8}{3}\right)^2$ $\frac{8}{3x}$ $\frac{1}{3x}$.

$$
\lim_{x \to \infty} \left(\frac{8}{3x}\right)^{\frac{1}{3x}} = \lim_{x \to \infty} e^{\left[\frac{\ln\left(\frac{8}{3x}\right)}{3x}\right]} = e^{\lim_{x \to \infty} \left[\frac{\ln\left(\frac{8}{3x}\right)}{3x}\right]}
$$

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ | 할 | X 9 Q Q

Example

Evaluate
$$
L = \lim_{n \to \infty} \left(\frac{8}{3n}\right)^{\frac{1}{3n}}
$$
.

Solution: We study a similar limit for the function $f(x) = \left(\frac{8}{3}\right)^2$ $\frac{8}{3x}$ $\frac{1}{3x}$.

$$
\lim_{x \to \infty} \left(\frac{8}{3x}\right)^{\frac{1}{3x}} = \lim_{x \to \infty} e^{\left[\frac{\ln\left(\frac{8}{3x}\right)}{3x}\right]} = e^{\lim_{x \to \infty} \left[\frac{\ln\left(\frac{8}{3x}\right)}{3x}\right]}
$$

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

Now, L'Hôpital's rule to find the limit in the exponent;

Example

Evaluate
$$
L = \lim_{n \to \infty} \left(\frac{8}{3n}\right)^{\frac{1}{3n}}
$$
.

Solution: We study a similar limit for the function $f(x) = \left(\frac{8}{3}\right)^2$ $\frac{8}{3x}$ $\frac{1}{3x}$.

$$
\lim_{x \to \infty} \left(\frac{8}{3x}\right)^{\frac{1}{3x}} = \lim_{x \to \infty} e^{\left[\frac{\ln\left(\frac{8}{3x}\right)}{3x}\right]} = e^{\lim_{x \to \infty} \left[\frac{\ln\left(\frac{8}{3x}\right)}{3x}\right]}
$$

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

Now, L'Hôpital's rule to find the limit in the exponent;

$$
\tilde{L} = \lim_{x \to \infty} \frac{\ln\left(\frac{8}{3x}\right)}{3x}
$$

Example

Evaluate
$$
L = \lim_{n \to \infty} \left(\frac{8}{3n}\right)^{\frac{1}{3n}}
$$
.

Solution: We study a similar limit for the function $f(x) = \left(\frac{8}{3}\right)^2$ $\frac{8}{3x}$ $\frac{1}{3x}$.

$$
\lim_{x \to \infty} \left(\frac{8}{3x}\right)^{\frac{1}{3x}} = \lim_{x \to \infty} e^{\left[\frac{\ln\left(\frac{8}{3x}\right)}{3x}\right]} = e^{\lim_{x \to \infty} \left[\frac{\ln\left(\frac{8}{3x}\right)}{3x}\right]}
$$

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

Now, L'Hôpital's rule to find the limit in the exponent;

$$
\tilde{L} = \lim_{x \to \infty} \frac{\ln\left(\frac{8}{3x}\right)}{3x} = \lim_{x \to \infty} \frac{\left(\frac{3x}{8} \frac{(-8)}{3x^2}\right)}{3}
$$

Example

Evaluate
$$
L = \lim_{n \to \infty} \left(\frac{8}{3n}\right)^{\frac{1}{3n}}
$$
.

Solution: We study a similar limit for the function $f(x) = \left(\frac{8}{3}\right)^2$ $\frac{8}{3x}$ $\frac{1}{3x}$.

$$
\lim_{x \to \infty} \left(\frac{8}{3x}\right)^{\frac{1}{3x}} = \lim_{x \to \infty} e^{\left[\frac{\ln\left(\frac{8}{3x}\right)}{3x}\right]} = e^{\lim_{x \to \infty} \left[\frac{\ln\left(\frac{8}{3x}\right)}{3x}\right]}
$$

Now, L'Hôpital's rule to find the limit in the exponent;

$$
\tilde{L} = \lim_{x \to \infty} \frac{\ln\left(\frac{8}{3x}\right)}{3x} = \lim_{x \to \infty} \frac{\left(\frac{3x}{8} \frac{(-8)}{3x^2}\right)}{3} = \lim_{x \to \infty} -\frac{1}{3x}
$$

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ (할) 1000

Example

Evaluate
$$
L = \lim_{n \to \infty} \left(\frac{8}{3n}\right)^{\frac{1}{3n}}
$$
.

Solution: We study a similar limit for the function $f(x) = \left(\frac{8}{3}\right)^2$ $\frac{8}{3x}$ $\frac{1}{3x}$.

$$
\lim_{x \to \infty} \left(\frac{8}{3x}\right)^{\frac{1}{3x}} = \lim_{x \to \infty} e^{\left[\frac{\ln\left(\frac{8}{3x}\right)}{3x}\right]} = e^{\lim_{x \to \infty} \left[\frac{\ln\left(\frac{8}{3x}\right)}{3x}\right]}
$$

Now, L'Hôpital's rule to find the limit in the exponent;

$$
\tilde{L}=\lim_{x\to\infty}\frac{\ln\bigl(\frac{8}{3x}\bigr)}{3x}=\lim_{x\to\infty}\frac{\bigl(\frac{3x}{8}\frac{(-8)}{3x^2}\bigr)}{3}=\lim_{x\to\infty}-\frac{1}{3x}=0.
$$

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

Example

Evaluate
$$
L = \lim_{n \to \infty} \left(\frac{8}{3n}\right)^{\frac{1}{3n}}
$$
.

Solution: We study a similar limit for the function $f(x) = \left(\frac{8}{3}\right)^2$ $\frac{8}{3x}$ $\frac{1}{3x}$.

$$
\lim_{x \to \infty} \left(\frac{8}{3x}\right)^{\frac{1}{3x}} = \lim_{x \to \infty} e^{\left[\frac{\ln\left(\frac{8}{3x}\right)}{3x}\right]} = e^{\lim_{x \to \infty} \left[\frac{\ln\left(\frac{8}{3x}\right)}{3x}\right]}
$$

Now, L'Hôpital's rule to find the limit in the exponent;

$$
\tilde{L}=\lim_{x\to\infty}\frac{\ln\bigl(\frac{8}{3x}\bigr)}{3x}=\lim_{x\to\infty}\frac{\bigl(\frac{3x}{8}\frac{(-8)}{3x^2}\bigr)}{3}=\lim_{x\to\infty}-\frac{1}{3x}=0.
$$

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ (할) 1000

Hence, $\lim_{x\to\infty} f(x) = e^0$

Example

Evaluate
$$
L = \lim_{n \to \infty} \left(\frac{8}{3n}\right)^{\frac{1}{3n}}
$$
.

Solution: We study a similar limit for the function $f(x) = \left(\frac{8}{3}\right)^2$ $\frac{8}{3x}$ $\frac{1}{3x}$.

$$
\lim_{x \to \infty} \left(\frac{8}{3x}\right)^{\frac{1}{3x}} = \lim_{x \to \infty} e^{\left[\frac{\ln\left(\frac{8}{3x}\right)}{3x}\right]} = e^{\lim_{x \to \infty} \left[\frac{\ln\left(\frac{8}{3x}\right)}{3x}\right]}
$$

Now, L'Hôpital's rule to find the limit in the exponent;

$$
\tilde{L}=\lim_{x\to\infty}\frac{\ln\bigl(\frac{8}{3x}\bigr)}{3x}=\lim_{x\to\infty}\frac{\bigl(\frac{3x}{8}\frac{(-8)}{3x^2}\bigr)}{3}=\lim_{x\to\infty}-\frac{1}{3x}=0.
$$

AD A REAKEN E VAN

Hence, $\lim_{x\to\infty} f(x) = e^0 = 1$,

Example

Evaluate
$$
L = \lim_{n \to \infty} \left(\frac{8}{3n}\right)^{\frac{1}{3n}}
$$
.

Solution: We study a similar limit for the function $f(x) = \left(\frac{8}{3}\right)^2$ $\frac{8}{3x}$ $\frac{1}{3x}$.

$$
\lim_{x \to \infty} \left(\frac{8}{3x}\right)^{\frac{1}{3x}} = \lim_{x \to \infty} e^{\left[\frac{\ln\left(\frac{8}{3x}\right)}{3x}\right]} = e^{\lim_{x \to \infty} \left[\frac{\ln\left(\frac{8}{3x}\right)}{3x}\right]}
$$

Now, L'Hôpital's rule to find the limit in the exponent;

$$
\tilde{L} = \lim_{x \to \infty} \frac{\ln\left(\frac{8}{3x}\right)}{3x} = \lim_{x \to \infty} \frac{\left(\frac{3x}{8} \frac{(-8)}{3x^2}\right)}{3} = \lim_{x \to \infty} -\frac{1}{3x} = 0.
$$

Hence, $\lim_{x \to \infty} f(x) = e^0 = 1$, therefore, $\lim_{n \to \infty} \left(\frac{8}{3n}\right)^{\frac{1}{3n}} = 1$.

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

Infinite series (Sect. 10.2)

- \blacktriangleright Series and partial sums.
- \blacktriangleright Geometric series.
- \blacktriangleright The *n*-term test for a divergent series.
- \triangleright Operations with series.
- \blacktriangleright Adding-deleting terms and re-indexing.

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

Infinite series (Sect. 10.2)

\triangleright Series and partial sums.

- \blacktriangleright Geometric series.
- \blacktriangleright The *n*-term test for a divergent series.
- \triangleright Operations with series.
- \blacktriangleright Adding-deleting terms and re-indexing.

KORK EX KEY KEY KORA

Definition

An infinite series is a sum of infinite terms,

```
a_1 + a_2 + a_3 + \cdots + a_n + \cdots
```
K ロ ▶ K @ ▶ K 할 X X 할 X → 할 X → 9 Q Q →

Definition

An infinite series is a sum of infinite terms,

$$
a_1 + a_2 + a_3 + \cdots + a_n + \cdots = \sum_{n=1}^{\infty} a_n
$$

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 | X 9 Q @

Definition

An infinite series is a sum of infinite terms,

$$
a_1 + a_2 + a_3 + \cdots + a_n + \cdots = \sum_{n=1}^{\infty} a_n
$$

K □ ▶ K @ ▶ K 할 X K 할 X (할) 10 Q Q Q

Remark: Any sequence $\{a_n\}_{n=1}^\infty$ defines the series $\sum^\infty a_n$. $n=1$

Definition

An infinite series is a sum of infinite terms,

$$
a_1 + a_2 + a_3 + \cdots + a_n + \cdots = \sum_{n=1}^{\infty} a_n.
$$

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

Remark: Any sequence $\{a_n\}_{n=1}^\infty$ defines the series $\sum^\infty a_n$. $n=1$

Example

The sequence $\Big\{ a_n = \frac{1}{2^n} \Big\}$ $\frac{1}{2^n}\Big\}_{n=1}^\infty$ $n=1$ defines the series

$$
\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n} + \cdots
$$

Definition

An infinite series is a sum of infinite terms,

$$
a_1 + a_2 + a_3 + \cdots + a_n + \cdots = \sum_{n=1}^{\infty} a_n.
$$

Remark: Any sequence $\{a_n\}_{n=1}^\infty$ defines the series $\sum^\infty a_n$. $n=1$

Example

The sequence
$$
\left\{ a_n = \frac{1}{2^n} \right\}_{n=1}^{\infty}
$$
 defines the series

$$
\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n} + \cdots
$$

This infinite sum makes sense, since

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ (할) 1000

Definition $\overline{\mathsf{G}}$ iven an infinite series $\sum_{n=1}^{\infty} a_n$, the sequence of partial sums of the $n=1$ series is the sequence $\{s_n\}$ given by $s_n = \sum^{n}$ $k=1$ a_k ,

Definition $\overline{\mathsf{G}}$ iven an infinite series $\sum_{n=1}^{\infty} a_n$, the sequence of partial sums of the $n=1$ series is the sequence $\{s_n\}$ given by $s_n = \sum_{k=1}^n a_k$, that is, $k=1$ $s_1 = a_1$ $s_2 = a_1 + a_2$ $s_3 = a_1 + a_2 + a_3$. . .

Definition $\overline{\mathsf{G}}$ iven an infinite series $\sum_{n=1}^{\infty} a_n$, the sequence of partial sums of the $n=1$ series is the sequence $\{s_n\}$ given by $s_n = \sum_{k=1}^n a_k$, that is, $k=1$ $s_1 = a_1$ $s_2 = a_1 + a_2$ $s_3 = a_1 + a_2 + a_3$. . . The series converges to L iff the sequence of partial sums $\{s_n\}$

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

converges to L,

Definition $\overline{\mathsf{G}}$ iven an infinite series $\sum_{n=1}^{\infty} a_n$, the sequence of partial sums of the $n=1$ series is the sequence $\{s_n\}$ given by $s_n = \sum_{k=1}^n a_k$, that is, $k=1$ $s_1 = a_1$ $s_2 = a_1 + a_2$ $s_3 = a_1 + a_2 + a_3$. . . The series converges to L iff the sequence of partial sums $\{s_n\}$ converges to L, and in this case we write $\sum^{\infty} a_n = L$. $n=1$

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

Definition $\overline{\mathsf{G}}$ iven an infinite series $\sum_{n=1}^{\infty} a_n$, the sequence of partial sums of the $n=1$ series is the sequence $\{s_n\}$ given by $s_n = \sum_{k=1}^n a_k$, that is, $k=1$ $s_1 = a_1$ $s_2 = a_1 + a_2$ $s_3 = a_1 + a_2 + a_3$. . . The series converges to L iff the sequence of partial sums $\{s_n\}$ converges to L, and in this case we write $\sum^{\infty} a_n = L$. The series diverges iff the sequence of partial sums $\{s_n\}^{n=1}$ diverges.

Remark: The series $a_1 + a_2 + a_3 + \cdots + a_n + \cdots$ can be denoted as

KB K K @ K K 통 K X 통 K 및 X Y Q Q @

Remark: The series $a_1 + a_2 + a_3 + \cdots + a_n + \cdots$ can be denoted as

KB K K @ K K 통 K X 통 K 및 X Y Q Q @

$$
\sum_{n=1}^{\infty} a_n, \qquad \sum_{k=1}^{\infty} a_k,
$$
Remark: The series $a_1 + a_2 + a_3 + \cdots + a_n + \cdots$ can be denoted as

$$
\sum_{n=1}^{\infty} a_n, \qquad \sum_{k=1}^{\infty} a_k, \qquad \sum a_n
$$

KB K K @ K K 통 K X 통 K 및 X Y Q Q @

Remark: The series $a_1 + a_2 + a_3 + \cdots + a_n + \cdots$ can be denoted as

$$
\sum_{n=1}^{\infty} a_n, \qquad \sum_{k=1}^{\infty} a_k, \qquad \sum a_n
$$

K □ ▶ K @ ▶ K 할 X K 할 X (할) 10 Q Q Q

Remark: The series $a_1 + a_2 + a_3 + \cdots + a_n + \cdots$ can be denoted as

$$
\sum_{n=1}^{\infty} a_n, \qquad \sum_{k=1}^{\infty} a_k, \qquad \sum a_n
$$

KB K K @ K K 통 K X 통 K 및 X Y Q Q @

Example
The series
$$
\sum_{n=1}^{\infty} \frac{1}{2^n}
$$
 converges to 1,

$$
\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} + \dots = 1
$$

Remark: The series $a_1 + a_2 + a_3 + \cdots + a_n + \cdots$ can be denoted as

$$
\sum_{n=1}^{\infty} a_n, \qquad \sum_{k=1}^{\infty} a_k, \qquad \sum a_n
$$

KB K K @ K K 통 K X 통 K 및 X Y Q Q @

Example
The series
$$
\sum_{n=1}^{\infty} \frac{1}{2^n}
$$
 converges to 1,

$$
\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} + \dots = 1
$$

Since $\{s_n\} \to 1$,

Remark: The series $a_1 + a_2 + a_3 + \cdots + a_n + \cdots$ can be denoted as

$$
\sum_{n=1}^{\infty} a_n, \qquad \sum_{k=1}^{\infty} a_k, \qquad \sum a_n
$$

Since $\{s_n\} \rightarrow 1$, as can be seen in the picture.

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

Example

$$
\blacktriangleright
$$
 The series
$$
\sum_{n=1}^{\infty} n
$$

Example

$$
\blacktriangleright
$$
 The series
$$
\sum_{n=1}^{\infty} n = 1 + 2 + 3 + \cdots + n + \cdots
$$

Example

• The series
$$
\sum_{n=1}^{\infty} n = 1 + 2 + 3 + \cdots + n + \cdots
$$
 diverges.

Example

► The series $\sum_{n=1}^{\infty} n = 1 + 2 + 3 + \cdots + n + \cdots$ diverges. $n=1$ Indeed, the sequence of partial sums diverges,

K □ ▶ K @ ▶ K 할 X K 할 X (할) 10 Q Q Q

Example

► The series $\sum_{n=1}^{\infty} n = 1 + 2 + 3 + \cdots + n + \cdots$ diverges. $n=1$ Indeed, the sequence of partial sums diverges,

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

 $s_1 = 1$,

Example

► The series $\sum_{n=1}^{\infty} n = 1 + 2 + 3 + \cdots + n + \cdots$ diverges. $n=1$ Indeed, the sequence of partial sums diverges,

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

 $s_1 = 1, \quad s_2 = 3,$

Example

► The series $\sum_{n=1}^{\infty} n = 1 + 2 + 3 + \cdots + n + \cdots$ diverges. $n=1$ Indeed, the sequence of partial sums diverges,

 $s_1 = 1$, $s_2 = 3$, $s_3 = 6$,

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

Example

► The series $\sum_{n=1}^{\infty} n = 1 + 2 + 3 + \cdots + n + \cdots$ diverges. $n=1$ Indeed, the sequence of partial sums diverges,

$$
s_1=1, \quad s_2=3, \quad s_3=6, \quad s_n=\sum_{k=1}^n k.
$$

K □ ▶ K @ ▶ K 할 X K 할 X (할) 10 Q Q Q

Example

► The series $\sum_{n=1}^{\infty} n = 1 + 2 + 3 + \cdots + n + \cdots$ diverges. Indeed, the sequence of partial sums diverges, $s_1 = 1, \quad s_2 = 3, \quad s_3 = 6, \quad s_n = \sum_{k=1}^{n} k.$ $k=1$ ► The series $\sum_{n=1}^{\infty}$ $n=1$ 1 n

K ロ X (日) X 제공 X 제공 X - 공 : X 이익(N)

Example

► The series $\sum_{n=1}^{\infty} n = 1 + 2 + 3 + \cdots + n + \cdots$ diverges. Indeed, the sequence of partial sums diverges, $s_1 = 1, \quad s_2 = 3, \quad s_3 = 6, \quad s_n = \sum_{k=1}^{n} k.$ $k=1$ ► The series $\sum_{n=1}^{\infty} \frac{1}{n}$ $n=1$ $\frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3}$ $\frac{1}{3} + \cdots + \frac{1}{n}$ $\frac{1}{n} + \cdots$

AD A REAKEN E YOUR

Example

► The series $\sum_{n=1}^{\infty} n = 1 + 2 + 3 + \cdots + n + \cdots$ diverges. Indeed, the sequence of partial sums diverges, $s_1 = 1, \quad s_2 = 3, \quad s_3 = 6, \quad s_n = \sum_{k=1}^{n} k.$ $k=1$ ► The series $\sum_{n=1}^{\infty} \frac{1}{n}$ n=1
harmonic series. $\frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3}$ $\frac{1}{3} + \cdots + \frac{1}{n}$ $\frac{1}{n} + \cdots$ is called the

Example

► The series $\sum_{n=1}^{\infty} n = 1 + 2 + 3 + \cdots + n + \cdots$ diverges. Indeed, the sequence of partial sums diverges, $s_1 = 1, \quad s_2 = 3, \quad s_3 = 6, \quad s_n = \sum_{k=1}^{n} k.$ $k=1$ ► The series $\sum_{n=1}^{\infty} \frac{1}{n}$ $n=1$
harmonic series. We will see that the harmonic series diverges. $\frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3}$ $\frac{1}{3} + \cdots + \frac{1}{n}$ $\frac{1}{n} + \cdots$ is called the

Example

► The series $\sum_{n=1}^{\infty} n = 1 + 2 + 3 + \cdots + n + \cdots$ diverges. Indeed, the sequence of partial sums diverges, $s_1 = 1, \quad s_2 = 3, \quad s_3 = 6, \quad s_n = \sum_{k=1}^{n} k.$ $k=1$ ► The series $\sum_{n=1}^{\infty} \frac{1}{n}$ $n=1$
harmonic series. We will see that the harmonic series diverges. $\frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3}$ $\frac{1}{3} + \cdots + \frac{1}{n}$ $\frac{1}{n} + \cdots$ is called the ► While the series $\sum_{n=1}^{\infty} \frac{(-1)^{(n+1)}}{n}$ $n=1$ n

Example

► The series $\sum_{n=1}^{\infty} n = 1 + 2 + 3 + \cdots + n + \cdots$ diverges. Indeed, the sequence of partial sums diverges, $s_1 = 1, \quad s_2 = 3, \quad s_3 = 6, \quad s_n = \sum_{k=1}^{n} k.$ $k=1$ ► The series $\sum_{n=1}^{\infty} \frac{1}{n}$ $n=1$
harmonic series. We will see that the harmonic series diverges. $\frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3}$ $\frac{1}{3} + \cdots + \frac{1}{n}$ $\frac{1}{n} + \cdots$ is called the ► While the series $\sum_{n=1}^{\infty} \frac{(-1)^{(n+1)}}{n}$ $n=1$ $\frac{1}{n} = 1 - \frac{1}{2}$ $\frac{1}{2} + \frac{1}{3}$ $\frac{1}{3} - \frac{1}{4}$ $\frac{1}{4} + \cdots$

Example

► The series $\sum_{n=1}^{\infty} n = 1 + 2 + 3 + \cdots + n + \cdots$ diverges. Indeed, the sequence of partial sums diverges, $s_1 = 1, \quad s_2 = 3, \quad s_3 = 6, \quad s_n = \sum_{k=1}^{n} k.$ $k=1$ ► The series $\sum_{n=1}^{\infty} \frac{1}{n}$ $n=1$
harmonic series. We will see that the harmonic series diverges. $\frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3}$ $\frac{1}{3} + \cdots + \frac{1}{n}$ $\frac{1}{n} + \cdots$ is called the ► While the series $\sum_{n=1}^{\infty} \frac{(-1)^{(n+1)}}{n}$ $n=1$ $\frac{1}{n} = 1 - \frac{1}{2}$ $\frac{1}{2} + \frac{1}{3}$ $\frac{1}{3} - \frac{1}{4}$ $\frac{1}{4} + \cdots$ converges.

Example

Evaluate the infinite series
$$
\frac{1}{2} + \frac{1}{(2)(3)} + \frac{1}{(3)(4)} + \frac{1}{(4)(5)} + \cdots
$$
.

Example

Evaluate the infinite series
$$
\frac{1}{2} + \frac{1}{(2)(3)} + \frac{1}{(3)(4)} + \frac{1}{(4)(5)} + \cdots
$$

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ | 할 | X 9 Q Q

Solution: We first find the general term a_n ,

Example

Evaluate the infinite series
$$
\frac{1}{2} + \frac{1}{(2)(3)} + \frac{1}{(3)(4)} + \frac{1}{(4)(5)} + \cdots
$$

Solution: We first find the general term a_n , that is,

$$
a_n=\frac{1}{n(n+1)},\qquad n=1,\cdots\infty.
$$

Example

Evaluate the infinite series
$$
\frac{1}{2} + \frac{1}{(2)(3)} + \frac{1}{(3)(4)} + \frac{1}{(4)(5)} + \cdots
$$

Solution: We first find the general term a_n , that is,

$$
a_n = \frac{1}{n(n+1)}, \qquad n = 1, \dots \infty.
$$

$$
\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \frac{1}{2} + \frac{1}{(2)(3)} + \frac{1}{(3)(4)} + \frac{1}{(4)(5)} + \dots
$$

Example

Evaluate the infinite series
$$
\frac{1}{2} + \frac{1}{(2)(3)} + \frac{1}{(3)(4)} + \frac{1}{(4)(5)} + \cdots
$$

Solution: We first find the general term a_n , that is,

$$
a_n = \frac{1}{n(n+1)}, \qquad n = 1, \dots \infty.
$$

$$
\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \frac{1}{2} + \frac{1}{(2)(3)} + \frac{1}{(3)(4)} + \frac{1}{(4)(5)} + \dots
$$

Partial fractions implies
$$
\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{(n+1)}\right).
$$

Example

Evaluate the infinite series
$$
\frac{1}{2} + \frac{1}{(2)(3)} + \frac{1}{(3)(4)} + \frac{1}{(4)(5)} + \cdots
$$

Solution: We first find the general term a_n , that is,

$$
a_n = \frac{1}{n(n+1)}, \qquad n = 1, \dots \infty.
$$

$$
\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \frac{1}{2} + \frac{1}{(2)(3)} + \frac{1}{(3)(4)} + \frac{1}{(4)(5)} + \dots
$$

Partial fractions implies
$$
\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{(n+1)}\right).
$$
 So,

Example

 $n=1$

Evaluate the infinite series
$$
\frac{1}{2} + \frac{1}{(2)(3)} + \frac{1}{(3)(4)} + \frac{1}{(4)(5)} + \cdots
$$

Solution: We first find the general term a_n , that is,

 $a_n = \frac{1}{n(n+1)}, \qquad n = 1, \cdots \infty.$ \sum^{∞} $n=1$ $\frac{1}{n(n+1)} = \frac{1}{2}$ $\frac{1}{2} + \frac{1}{(2)(3)} + \frac{1}{(3)(4)} + \frac{1}{(4)(5)} + \cdots$ Partial fractions implies $\sum_{n=0}^{\infty}$ $n=1$ $\frac{1}{n(n+1)} =$ \sum^{∞} $n=1$ $\left(\frac{1}{2}\right)$ $\frac{1}{n} - \frac{1}{(n+1)}$ $(n+1)$. So, \sum^{∞} 1 $n(n+1)$

4 0 > 4 4 + 4 3 + 4 3 + 5 + 9 4 0 +

Example

Evaluate the infinite series
$$
\frac{1}{2} + \frac{1}{(2)(3)} + \frac{1}{(3)(4)} + \frac{1}{(4)(5)} + \cdots
$$

Solution: We first find the general term a_n , that is,

 $a_n = \frac{1}{n(n+1)}, \qquad n = 1, \cdots \infty.$ \sum^{∞} $n=1$ $\frac{1}{n(n+1)} = \frac{1}{2}$ $\frac{1}{2} + \frac{1}{(2)(3)} + \frac{1}{(3)(4)} + \frac{1}{(4)(5)} + \cdots$ Partial fractions implies $\sum_{n=0}^{\infty}$ $n=1$ $\frac{1}{n(n+1)} =$ \sum^{∞} $n=1$ $\left(\frac{1}{2}\right)$ $\frac{1}{n} - \frac{1}{(n+1)}$ $(n+1)$. So, \sum^{∞} $n=1$ $\frac{1}{n(n+1)} = 1 + \frac{1}{2} + \frac{1}{3}$ $\frac{1}{3} + \cdots$

Example

Evaluate the infinite series
$$
\frac{1}{2} + \frac{1}{(2)(3)} + \frac{1}{(3)(4)} + \frac{1}{(4)(5)} + \cdots
$$

Solution: We first find the general term a_n , that is,

 $a_n = \frac{1}{n(n+1)}, \qquad n = 1, \cdots \infty.$ \sum^{∞} $n=1$ $\frac{1}{n(n+1)} = \frac{1}{2}$ $\frac{1}{2} + \frac{1}{(2)(3)} + \frac{1}{(3)(4)} + \frac{1}{(4)(5)} + \cdots$ Partial fractions implies $\sum_{n=0}^{\infty}$ $n=1$ $\frac{1}{n(n+1)} =$ \sum^{∞} $n=1$ $\left(\frac{1}{2}\right)$ $\frac{1}{n} - \frac{1}{(n+1)}$ $(n+1)$. So, \sum^{∞} $n=1$ $\frac{1}{n(n+1)} = 1 + \frac{1}{2} + \frac{1}{3}$ $\frac{1}{3} + \cdots - \frac{1}{2}$ $\frac{1}{2} - \frac{1}{3}$ $\frac{1}{3} - \frac{1}{4}$ $\frac{1}{4} - \cdots$

A DIA K PIA K E A SHA K H A K A K A SHA K E A SHA K A SHA

Example

Evaluate the infinite series
$$
\frac{1}{2} + \frac{1}{(2)(3)} + \frac{1}{(3)(4)} + \frac{1}{(4)(5)} + \cdots
$$

Solution: We first find the general term a_n , that is,

$$
a_n = \frac{1}{n(n+1)}, \qquad n = 1, \dots \infty.
$$

$$
\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \frac{1}{2} + \frac{1}{(2)(3)} + \frac{1}{(3)(4)} + \frac{1}{(4)(5)} + \dots
$$

Partial fractions implies
$$
\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{(n+1)}\right).
$$
 So,

$$
\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1 + \frac{1}{2} + \frac{1}{3} + \dots - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} - \dots = 1.
$$

Example

Evaluate the infinite series
$$
\frac{1}{2} + \frac{1}{(2)(3)} + \frac{1}{(3)(4)} + \frac{1}{(4)(5)} + \cdots
$$

Solution: We first find the general term a_n , that is,

 $a_n = \frac{1}{n(n+1)}, \qquad n = 1, \cdots \infty.$ \sum^{∞} $n=1$ $\frac{1}{n(n+1)} = \frac{1}{2}$ $\frac{1}{2} + \frac{1}{(2)(3)} + \frac{1}{(3)(4)} + \frac{1}{(4)(5)} + \cdots$ Partial fractions implies $\sum_{n=0}^{\infty}$ $n=1$ $\frac{1}{n(n+1)} =$ \sum^{∞} $n=1$ $\left(\frac{1}{2}\right)$ $\frac{1}{n} - \frac{1}{(n+1)}$ $(n+1)$. So, \sum^{∞} $n=1$ $\frac{1}{n(n+1)} = 1 + \frac{1}{2} + \frac{1}{3}$ $\frac{1}{3} + \cdots - \frac{1}{2}$ $\frac{1}{2} - \frac{1}{3}$ $\frac{1}{3} - \frac{1}{4}$ $\frac{1}{4} - \cdots = 1.$ We conclude: $\sum_{n=0}^{\infty}$ $n=1$ $\frac{1}{n(n+1)} = 1.$

Infinite series (Sect. 10.2)

- \blacktriangleright Series and partial sums.
- \blacktriangleright Geometric series.
- \blacktriangleright The *n*-term test for a divergent series.
- \triangleright Operations with series.
- \blacktriangleright Adding-deleting terms and re-indexing.

KORK EX KEY KEY KORA

Definition A geometric series is a series of the form

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ (할) 수 있습니다

 \sum^{∞} $n=0$ ar^n

Definition A geometric series is a series of the form

$$
\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + ar^3 + \cdots
$$

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ | 할 | X 9 Q Q

where a and r are real numbers.

Definition

A geometric series is a series of the form

$$
\sum_{n=0}^{\infty} a r^n = a + ar + ar^2 + ar^3 + \cdots.
$$

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

where a and r are real numbers.

Example

The case $a=1$, and ratio $r=\frac{1}{2}$ $\frac{1}{2}$ is the geometric series

$$
\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n
$$

Definition

A geometric series is a series of the form

$$
\sum_{n=0}^{\infty} a r^n = a + ar + ar^2 + ar^3 + \cdots.
$$

where a and r are real numbers.

Example

The case $a=1$, and ratio $r=\frac{1}{2}$ $\frac{1}{2}$ is the geometric series

$$
\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots.
$$

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^
Definition

A geometric series is a series of the form

$$
\sum_{n=0}^{\infty} a r^n = a + ar + ar^2 + ar^3 + \cdots
$$

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

where a and r are real numbers.

Example

The case $a=1$, and ratio $r=\frac{1}{2}$ $\frac{1}{2}$ is the geometric series

$$
\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots
$$

We have seen $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots = 1$,

Definition

A geometric series is a series of the form

$$
\sum_{n=0}^{\infty} a r^n = a + ar + ar^2 + ar^3 + \cdots
$$

where a and r are real numbers.

Example

The case $a=1$, and ratio $r=\frac{1}{2}$ $\frac{1}{2}$ is the geometric series

$$
\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots
$$

We have seen $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots = 1$, so $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = 2$

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

Theorem The sectance of $\sum_{n=1}^{\infty} a r^n$ has ratio $|r| < 1$, then converges, $n=0$ \sum^{∞} $ar^n = \frac{a}{1}$ $\frac{1}{1-r}$.

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

 $n=0$

Theorem The sectance of $\sum_{n=1}^{\infty} a r^n$ has ratio $|r| < 1$, then converges, $n=0$ \sum^{∞} $n=0$ $ar^n = \frac{a}{1}$ $\frac{1}{1-r}$.

AD A REAKEN E YOUR

Proof: Multiply any partial sum s_n by $(1 - r)$,

Theorem The sectance of $\sum_{n=1}^{\infty} a r^n$ has ratio $|r| < 1$, then converges, $n=0$ \sum^{∞} $n=0$ $ar^n = \frac{a}{1}$ $\frac{1}{1-r}$.

Proof: Multiply any partial sum s_n by $(1 - r)$, that is,

$$
(1 - r)s_n = a(1 - r)(1 + r + r^2 + \cdots + r^n)
$$

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

Theorem The sectance of $\sum_{n=1}^{\infty} a r^n$ has ratio $|r| < 1$, then converges, $n=0$ \sum^{∞} $n=0$ $ar^n = \frac{a}{1}$ $\frac{1}{1-r}$.

Proof: Multiply any partial sum s_n by $(1 - r)$, that is,

$$
(1 - r)s_n = a(1 - r)(1 + r + r^2 + \cdots + r^n)
$$

 $(1 - r)s_n = a(1 + r + r^2 + \cdots + r^n) - a(r + r^2 + r^3 + \cdots + r^{n+1})$

AD A REAKEN E YOUR

Theorem The sectance of $\sum_{n=1}^{\infty} a r^n$ has ratio $|r| < 1$, then converges, $n=0$ \sum^{∞} $n=0$ $ar^n = \frac{a}{1}$ $\frac{1}{1-r}$.

Proof: Multiply any partial sum s_n by $(1 - r)$, that is,

$$
(1 - r)s_n = a(1 - r)(1 + r + r^2 + \cdots + r^n)
$$

$$
(1 - r)s_n = a(1 + r + r^2 + \dots + r^n) - a(r + r^2 + r^3 + \dots + r^{n+1})
$$

$$
(1 - r)s_n = a(1 - r^{n+1})
$$

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

Theorem The sectance of $\sum_{n=1}^{\infty} a r^n$ has ratio $|r| < 1$, then converges, $n=0$ \sum^{∞} $n=0$ $ar^n = \frac{a}{1}$ $\frac{1}{1-r}$.

Proof: Multiply any partial sum s_n by $(1 - r)$, that is,

$$
(1 - r)s_n = a(1 - r)(1 + r + r^2 + \cdots + r^n)
$$

$$
(1 - r)s_n = a(1 + r + r^2 + \dots + r^n) - a(r + r^2 + r^3 + \dots + r^{n+1})
$$

$$
(1 - r)s_n = a(1 - r^{n+1}) \Rightarrow s_n = \frac{a(1 - r^{n+1})}{(1 - r)}.
$$

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

Theorem The sectance of $\sum_{n=1}^{\infty} a r^n$ has ratio $|r| < 1$, then converges, $n=0$ \sum^{∞} $n=0$ $ar^n = \frac{a}{1}$ $\frac{1}{1-r}$.

Proof: Multiply any partial sum s_n by $(1 - r)$, that is,

$$
(1 - r)s_n = a(1 - r)(1 + r + r^2 + \cdots + r^n)
$$

$$
(1 - r)s_n = a(1 + r + r^2 + \dots + r^n) - a(r + r^2 + r^3 + \dots + r^{n+1})
$$

$$
(1 - r)s_n = a(1 - r^{n+1}) \Rightarrow s_n = \frac{a(1 - r^{n+1})}{(1 - r)}.
$$

AD A REAKEN E YOUR

Since $|r| < 1$,

Theorem The sectance of $\sum_{n=1}^{\infty} a r^n$ has ratio $|r| < 1$, then converges, $n=0$ \sum^{∞} $n=0$ $ar^n = \frac{a}{1}$ $\frac{1}{1-r}$.

Proof: Multiply any partial sum s_n by $(1 - r)$, that is,

$$
(1 - r)s_n = a(1 - r)(1 + r + r^2 + \cdots + r^n)
$$

$$
(1 - r)s_n = a(1 + r + r^2 + \dots + r^n) - a(r + r^2 + r^3 + \dots + r^{n+1})
$$

$$
(1 - r)s_n = a(1 - r^{n+1}) \Rightarrow s_n = \frac{a(1 - r^{n+1})}{(1 - r)}.
$$

AD A REAKEN E YOUR

Since $|r| < 1$, then $r^{n+1} \rightarrow 0$.

Example

Evaluate the infinite series $\sum_{n=0}^{\infty}$

 $n=0$ 1 $\frac{1}{2^n}$.

Example

Evaluate the infinite series $\sum_{n=0}^{\infty}$ $n=0$

Solution: Recall the picture says
$$
\sum_{n=0}^{\infty} \frac{1}{2^n} = 2
$$
.

1 $\frac{1}{2^n}$.

Example

Evaluate the infinite series $\sum_{n=0}^{\infty}$ $n=0$

Solution: Recall the picture says
$$
\sum_{n=0}^{\infty} \frac{1}{2^n} = 2
$$
.

We now use the Theorem above,
$$
\sum_{n=0}^{\infty} a r^n = \frac{a}{1-r}
$$

1 $\frac{1}{2^n}$.

Example

Evaluate the infinite series $\sum_{n=0}^{\infty}$ $n=0$ $\frac{1}{2^n}$.

Solution: Recall the picture says
$$
\sum_{n=0}^{\infty} \frac{1}{2^n} = 2.
$$

We now use the Theorem above,
$$
\sum_{n=0}^{\infty} a r^n = \frac{a}{1-r},
$$
 for $a = 1$ and $r = \frac{1}{2}$.

1

Example

Evaluate the infinite series $\sum_{n=0}^{\infty}$ $n=0$

Solution: Recall the picture says
$$
\sum_{n=0}^{\infty} \frac{1}{2^n} = 2
$$
.

We now use the Theorem above,
$$
\sum_{n=0}^{\infty} a r^n = \frac{a}{1-r},
$$
 for $a = 1$ and $r = \frac{1}{2}$.

1 $\frac{1}{2^n}$.

$$
\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = \frac{1}{1-\frac{1}{2}}
$$

Example

Evaluate the infinite series $\sum_{n=0}^{\infty}$ $n=0$ $\frac{1}{2^n}$.

Solution: Recall the picture says
$$
\sum_{n=0}^{\infty} \frac{1}{2^n} = 2.
$$

1

We now use the Theorem above,
$$
\sum_{n=0}^{\infty} a r^n = \frac{a}{1-r}
$$
,
for $a = 1$ and $r = \frac{1}{2}$.

$$
\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = \frac{1}{1-\frac{1}{2}} = \frac{1}{\left(\frac{2-1}{2}\right)}
$$

Example

Evaluate the infinite series $\sum_{n=0}^{\infty}$ $n=0$

 $n=0$

2

Solution: Recall the picture says
$$
\sum_{n=0}^{\infty} \frac{1}{2^n} = 2
$$
.

2

We now use the Theorem above,
$$
\sum_{n=0}^{\infty} a r^n = \frac{a}{1-r},
$$

for $a = 1$ and $r = \frac{1}{2}$.

$$
\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = \frac{1}{1-\frac{1}{2}} = \frac{1}{\left(\frac{2-1}{2}\right)} \implies \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = 2.
$$

 $n=0$

2

K □ ▶ K @ ▶ K 할 X K 할 X (할) 10 Q Q Q

1 $\frac{1}{2^n}$.

Example

Evaluate the infinite sum

$$
\sum_{n=1}^{\infty}(-1)^{(n+1)}\frac{3}{4^n}.
$$

Example

Evaluate the infinite sum

$$
\sum_{n=1}^{\infty}(-1)^{(n+1)}\frac{3}{4^n}.
$$

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 | X 9 Q @

Solution: This is a geometric series,

Example

Evaluate the infinite sum
$$
\sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{3}{4^n}.
$$

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ | 할 | X 9 Q Q

Solution: This is a geometric series, since

$$
\sum_{n=1}^{\infty}(-1)^{(n+1)}\frac{3}{4^n}
$$

Example

Evaluate the infinite sum
$$
\sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{3}{4^n}.
$$

Solution: This is a geometric series, since

$$
\sum_{n=1}^{\infty}(-1)^{(n+1)}\frac{3}{4^n}=\sum_{n=1}^{\infty}(-1)^n\frac{(-3)}{4^n}
$$

Example

Evaluate the infinite sum
$$
\sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{3}{4^n}.
$$

Solution: This is a geometric series, since

$$
\sum_{n=1}^{\infty}(-1)^{(n+1)}\frac{3}{4^n}=\sum_{n=1}^{\infty}(-1)^n\frac{(-3)}{4^n}=\sum_{n=1}^{\infty}(-3)\left(-\frac{1}{4}\right)^n.
$$

Example

Evaluate the infinite sum
$$
\sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{3}{4^n}.
$$

Solution: This is a geometric series, since

$$
\sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{3}{4^n} = \sum_{n=1}^{\infty} (-1)^n \frac{(-3)}{4^n} = \sum_{n=1}^{\infty} (-3) \left(-\frac{1}{4}\right)^n.
$$

Hence $a = -3$ and $r = -\frac{1}{4}$.

K ロ X イロ X K ミ X K ミ X ミ → S V Q Q Q

Example

Evaluate the infinite sum
$$
\sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{3}{4^n}.
$$

Solution: This is a geometric series, since

$$
\sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{3}{4^n} = \sum_{n=1}^{\infty} (-1)^n \frac{(-3)}{4^n} = \sum_{n=1}^{\infty} (-3) \left(-\frac{1}{4}\right)^n.
$$

Hence $a = -3$ and $r = -\frac{1}{4}$. The Theorem above implies,

$$
\sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{3}{4^n}
$$

Example

Evaluate the infinite sum
$$
\sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{3}{4^n}.
$$

Solution: This is a geometric series, since

$$
\sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{3}{4^n} = \sum_{n=1}^{\infty} (-1)^n \frac{(-3)}{4^n} = \sum_{n=1}^{\infty} (-3) \left(-\frac{1}{4}\right)^n.
$$

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ | 할 | X 9 Q Q

Hence $a = -3$ and $r = -\frac{1}{4}$ $\frac{1}{4}$. The Theorem above implies,
4

$$
\sum_{n=1}^{\infty}(-1)^{(n+1)}\frac{3}{4^n}=\sum_{n=0}^{\infty}(-1)^{(n+1)}\frac{3}{4^n}-(-3)
$$

Example

Evaluate the infinite sum
$$
\sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{3}{4^n}.
$$

Solution: This is a geometric series, since

$$
\sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{3}{4^n} = \sum_{n=1}^{\infty} (-1)^n \frac{(-3)}{4^n} = \sum_{n=1}^{\infty} (-3) \left(-\frac{1}{4}\right)^n.
$$

Hence $a = -3$ and $r = -\frac{1}{4}$. The Theorem above implies,
 ∞

$$
\sum_{n=1}^{\infty}(-1)^{(n+1)}\frac{3}{4^n}=\sum_{n=0}^{\infty}(-1)^{(n+1)}\frac{3}{4^n}-(-3)=\frac{(-3)}{(1+\frac{1}{4})}+3
$$

Example

Evaluate the infinite sum
$$
\sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{3}{4^n}.
$$

Solution: This is a geometric series, since

$$
\sum_{n=1}^{\infty}(-1)^{(n+1)}\frac{3}{4^n}=\sum_{n=1}^{\infty}(-1)^n\frac{(-3)}{4^n}=\sum_{n=1}^{\infty}(-3)\left(-\frac{1}{4}\right)^n.
$$

Hence $a = -3$ and $r = -\frac{1}{4}$ $\frac{1}{4}$. The Theorem above implies,
4

$$
\sum_{n=1}^{\infty}(-1)^{(n+1)}\frac{3}{4^n}=\sum_{n=0}^{\infty}(-1)^{(n+1)}\frac{3}{4^n}-(-3)=\frac{(-3)}{(1+\frac{1}{4})}+3
$$

$$
\sum_{n=1}^{\infty}(-1)^{(n+1)}\frac{3}{4^n}=-\frac{3}{\left(\frac{4+1}{4}\right)}+3,
$$

Example

Evaluate the infinite sum
$$
\sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{3}{4^n}.
$$

Solution: This is a geometric series, since

$$
\sum_{n=1}^{\infty}(-1)^{(n+1)}\frac{3}{4^n}=\sum_{n=1}^{\infty}(-1)^n\frac{(-3)}{4^n}=\sum_{n=1}^{\infty}(-3)\left(-\frac{1}{4}\right)^n.
$$

Hence $a = -3$ and $r = -\frac{1}{4}$ 4 . The Theorem above implies,

$$
\sum_{n=1}^{\infty}(-1)^{(n+1)}\frac{3}{4^n}=\sum_{n=0}^{\infty}(-1)^{(n+1)}\frac{3}{4^n}-(-3)=\frac{(-3)}{(1+\frac{1}{4})}+3
$$

$$
\sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{3}{4^n} = -\frac{3}{\left(\frac{4+1}{4}\right)} + 3, \text{ then } \sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{3}{4^n} = -\frac{12}{5} + 3.
$$

Infinite series (Sect. 10.2)

- \blacktriangleright Series and partial sums.
- \blacktriangleright Geometric series.
- \blacktriangleright The *n*-term test for a divergent series.
- \triangleright Operations with series.
- \blacktriangleright Adding-deleting terms and re-indexing.

KORK EX KEY KEY KORA

Theorem If the series $\sum_{n=1}^{\infty} a_n$ converges, then $a_n \to 0$. $n=1$

K □ ▶ K @ ▶ K 할 X K 할 X (할) 2000

Theorem If the series $\sum_{n=1}^{\infty} a_n$ converges, then $a_n \to 0$. $n=1$

Remark: This result is useful to find divergent series.

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

Theorem If the series $\sum_{n=1}^{\infty} a_n$ converges, then $a_n \to 0$. $n=1$ Remark: This result is useful to find divergent series. Remark: If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ $n=1$ a_n diverges.

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

Theorem If the series $\sum_{n=1}^{\infty} a_n$ converges, then $a_n \to 0$. $n=1$ Remark: This result is useful to find divergent series. Remark: If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ $n=1$ a_n diverges. Example

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

$$
\blacktriangleright \sum_{n=1}^{\infty} n \text{ diverges},
$$

 $n=1$

Theorem If the series $\sum_{n=1}^{\infty} a_n$ converges, then $a_n \to 0$. $n=1$ Remark: This result is useful to find divergent series. Remark: If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ $n=1$ a_n diverges. Example \blacktriangleright $\sum_{n=1}^{\infty} n$ diverges, since $n \to \infty$.

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

Theorem If the series $\sum_{n=1}^{\infty} a_n$ converges, then $a_n \to 0$. $n=1$ Remark: This result is useful to find divergent series. Remark: If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ $n=1$ a_n diverges. Example

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

►
$$
\sum_{n=1}^{\infty} n \text{ diverges, since } n \to \infty.
$$

▶
$$
\sum_{n=1}^{\infty} \frac{n}{n+1} \text{ diverges,}
$$

Theorem If the series $\sum_{n=1}^{\infty} a_n$ converges, then $a_n \to 0$. $n=1$ Remark: This result is useful to find divergent series. Remark: If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ $n=1$ a_n diverges. Example \blacktriangleright $\sum_{n=1}^{\infty} n$ diverges, since $n \to \infty$. $n=1$ ∞

$$
\blacktriangleright \sum_{n=1}^{\infty} \frac{n}{n+1} \text{ diverges, since } \frac{n}{n+1} \to 1 \neq 0.
$$

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^
The *n*-term test for a divergent series

Theorem If the series $\sum_{n=1}^{\infty} a_n$ converges, then $a_n \to 0$. $n=1$ Remark: This result is useful to find divergent series. Remark: If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ $n=1$ a_n diverges. Example \blacktriangleright $\sum_{n=1}^{\infty} n$ diverges, since $n \to \infty$. $n=1$ \sum^{∞} $n=1$ n $\frac{n}{n+1}$ diverges, since $\frac{n}{n+1} \to 1 \neq 0$. $\sum_{n=1}^{\infty} (-1)^n$ diverges, $n=1$

The *n*-term test for a divergent series

Theorem If the series $\sum_{n=1}^{\infty} a_n$ converges, then $a_n \to 0$. $n=1$ Remark: This result is useful to find divergent series. Remark: If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ $n=1$ a_n diverges. Example \blacktriangleright $\sum_{n=1}^{\infty} n$ diverges, since $n \to \infty$. $n=1$ \sum^{∞} $n=1$ n $\frac{n}{n+1}$ diverges, since $\frac{n}{n+1} \to 1 \neq 0$. ► $\sum_{n=0}^{\infty} (-1)^n$ diverges, since $\lim_{n\to\infty} (-1)^n$ does not exist. $n=1$

4 D > 4 P > 4 E > 4 E > E + 9 Q O

Infinite series (Sect. 10.2)

- \blacktriangleright Series and partial sums.
- \blacktriangleright Geometric series.
- \blacktriangleright The *n*-term test for a divergent series.
- \triangleright Operations with series.
- \blacktriangleright Adding-deleting terms and re-indexing.

KORK EX KEY KEY KORA

Operations with series

Remark: Additions of convergent series define convergent series.

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ | 할 | X 9 Q Q

Operations with series

Remark: Additions of convergent series define convergent series.

AD A REAKEN E YOUR

Theorem If the series $\sum_{n=1}^{\infty}$ $n=1$ $a_n = A$ and $\sum_{n=1}^{\infty}$ $n=1$ $b_n = B$, then $\sum_{n=1}^{\infty} (a_n + b_n) = A + B;$ $n=1$ $\sum_{n=1}^{\infty} (a_n - b_n) = A - B;$ $n=1$ \sum^{∞} $n=1$ $ka_n = kA$.

Infinite series (Sect. 10.2)

- \blacktriangleright Series and partial sums.
- \blacktriangleright Geometric series.
- \blacktriangleright The *n*-term test for a divergent series.
- \triangleright Operations with series.
- \blacktriangleright Adding-deleting terms and re-indexing.

KORK EX KEY KEY KORA

Remarks:

 \triangleright Adding or deleting a finite number of terms to series does not change the series convergence or divergence.

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

Remarks:

 \triangleright Adding or deleting a finite number of terms to series does not change the series convergence or divergence.

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

Example:
$$
\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \sum_{n=4}^{\infty} \frac{1}{2^n}.
$$

Remarks:

▶ Adding or deleting a finite number of terms to series does not change the series convergence or divergence.

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

Example:
$$
\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \sum_{n=4}^{\infty} \frac{1}{2^n}.
$$

Remarks:

▶ Adding or deleting a finite number of terms to series does not change the series convergence or divergence.

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

Example:
$$
\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \sum_{n=4}^{\infty} \frac{1}{2^n}.
$$

Example:
$$
\sum_{n=1}^{\infty} a_n
$$

Remarks:

▶ Adding or deleting a finite number of terms to series does not change the series convergence or divergence.

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

Example:
$$
\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \sum_{n=4}^{\infty} \frac{1}{2^n}.
$$

Example:
$$
\sum_{n=1}^{\infty} a_n = \sum_{\ell=1}^{\infty} a_\ell
$$

Remarks:

▶ Adding or deleting a finite number of terms to series does not change the series convergence or divergence.

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

Example:
$$
\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \sum_{n=4}^{\infty} \frac{1}{2^n}.
$$

Example:
$$
\sum_{n=1}^{\infty} a_n = \sum_{\ell=1}^{\infty} a_\ell = \sum_{k=7}^{\infty} a_{k-6}.
$$

Remarks:

▶ Adding or deleting a finite number of terms to series does not change the series convergence or divergence.

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

Example:
$$
\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \sum_{n=4}^{\infty} \frac{1}{2^n}.
$$

Example:
$$
\sum_{n=1}^{\infty} a_n = \sum_{\ell=1}^{\infty} a_\ell = \sum_{k=7}^{\infty} a_{k-6}.
$$

Example:
$$
\sum_{n=0}^{\infty} \frac{1}{2^n}
$$

Remarks:

▶ Adding or deleting a finite number of terms to series does not change the series convergence or divergence.

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

Example:
$$
\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \sum_{n=4}^{\infty} \frac{1}{2^n}.
$$

Example:
$$
\sum_{n=1}^{\infty} a_n = \sum_{\ell=1}^{\infty} a_{\ell} = \sum_{k=7}^{\infty} a_{k-6}.
$$

Example: $\sum_{n=0}^{\infty} \frac{1}{2^n} = \sum_{k=8}^{\infty} \frac{1}{2^{(k-7)}}$

Remarks:

▶ Adding or deleting a finite number of terms to series does not change the series convergence or divergence.

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

Example:
$$
\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \sum_{n=4}^{\infty} \frac{1}{2^n}.
$$

Example:
$$
\sum_{n=1}^{\infty} a_n = \sum_{\ell=1}^{\infty} a_{\ell} = \sum_{k=7}^{\infty} a_{k-6}.
$$

Example:
$$
\sum_{n=0}^{\infty} \frac{1}{2^n} = \sum_{k=8}^{\infty} \frac{1}{2^{(k-7)}} = \sum_{k=8}^{\infty} \frac{2^7}{2^k}.
$$

The integral test (Sect. 10.3)

 \blacktriangleright Review: Bounded and monotonic sequences.

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

- \blacktriangleright Application: The harmonic series.
- \blacktriangleright Testing with an integral.
- \blacktriangleright Error estimation in the integral test.

The integral test (Sect. 10.3)

\triangleright Review: Bounded and monotonic sequences.

KORK EX KEY KEY KORA

- \blacktriangleright Application: The harmonic series.
- \blacktriangleright Testing with an integral.
- \blacktriangleright Error estimation in the integral test.

Definition

A sequence $\{a_n\}$ is bounded above iff there is $M \in \mathbb{R}$ such that

 $a_n \leqslant M$ for all $n \geqslant 1$.

K □ ▶ K @ ▶ K 할 X K 할 X (할) 2000

Definition

A sequence $\{a_n\}$ is bounded above iff there is $M \in \mathbb{R}$ such that

 $a_n \leqslant M$ for all $n \geqslant 1$.

The sequence $\{a_n\}$ is bounded below iff there is $m \in \mathbb{R}$ such that

 $m \leq a_n$ for all $n \geq 1$.

Definition A sequence $\{a_n\}$ is bounded above iff there is $M \in \mathbb{R}$ such that

 $a_n \leqslant M$ for all $n \geqslant 1$.

The sequence $\{a_n\}$ is bounded below iff there is $m \in \mathbb{R}$ such that

 $m \le a_n$ for all $n \ge 1$.

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

A sequence is bounded iff it is bounded above and below.

Definition A sequence $\{a_n\}$ is bounded above iff there is $M \in \mathbb{R}$ such that

 $a_n \leqslant M$ for all $n \geqslant 1$.

The sequence $\{a_n\}$ is bounded below iff there is $m \in \mathbb{R}$ such that

 $m \le a_n$ for all $n \ge 1$.

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

A sequence is bounded iff it is bounded above and below.

$$
\blacktriangleright a_n = \frac{1}{n} \text{ is bounded,}
$$

Definition A sequence $\{a_n\}$ is bounded above iff there is $M \in \mathbb{R}$ such that

 $a_n \leqslant M$ for all $n \geqslant 1$.

The sequence $\{a_n\}$ is bounded below iff there is $m \in \mathbb{R}$ such that

 $m \le a_n$ for all $n \ge 1$.

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

A sequence is bounded iff it is bounded above and below.

$$
\blacktriangleright a_n = \frac{1}{n} \text{ is bounded, since } 0 < \frac{1}{n} \leq 1.
$$

Definition A sequence $\{a_n\}$ is bounded above iff there is $M \in \mathbb{R}$ such that

 $a_n \leqslant M$ for all $n \geqslant 1$.

The sequence $\{a_n\}$ is bounded below iff there is $m \in \mathbb{R}$ such that

 $m \le a_n$ for all $n \ge 1$.

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

A sequence is bounded iff it is bounded above and below.

▶
$$
a_n = \frac{1}{n}
$$
 is bounded, since $0 < \frac{1}{n} \leq 1$.
\n▶ $a_n = (-1)^n$ is bounded,

Definition A sequence $\{a_n\}$ is bounded above iff there is $M \in \mathbb{R}$ such that

 $a_n \leqslant M$ for all $n \geqslant 1$.

The sequence $\{a_n\}$ is bounded below iff there is $m \in \mathbb{R}$ such that

 $m \le a_n$ for all $n \ge 1$.

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

A sequence is bounded iff it is bounded above and below.

\n- $$
a_n = \frac{1}{n}
$$
 is bounded, since $0 < \frac{1}{n} \leq 1$.
\n- $a_n = (-1)^n$ is bounded, since $-1 \leq (-1)^n \leq 1$.
\n

Definition

A sequence $\{a_n\}$ is increasing iff $a_n < a_{n+1}$.

Definition

- A sequence $\{a_n\}$ is increasing iff $a_n < a_{n+1}$.
- A sequence $\{a_n\}$ is non-decreasing iff $a_n \leq a_{n+1}$.

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

Definition

- A sequence $\{a_n\}$ is increasing iff $a_n < a_{n+1}$.
- A sequence $\{a_n\}$ is non-decreasing iff $a_n \le a_{n+1}$.

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

A sequence $\{a_n\}$ is decreasing iff $a_n > a_{n+1}$.

Definition

- A sequence $\{a_n\}$ is increasing iff $a_n < a_{n+1}$.
- A sequence $\{a_n\}$ is non-decreasing iff $a_n \le a_{n+1}$.
- A sequence $\{a_n\}$ is decreasing iff $a_n > a_{n+1}$.
- A sequence $\{a_n\}$ is non-increasing iff $a_n \ge a_{n+1}$.

Definition

- A sequence $\{a_n\}$ is increasing iff $a_n < a_{n+1}$.
- A sequence $\{a_n\}$ is non-decreasing iff $a_n \le a_{n+1}$.
- A sequence $\{a_n\}$ is decreasing iff $a_n > a_{n+1}$.
- A sequence $\{a_n\}$ is non-increasing iff $a_n \ge a_{n+1}$.
- \triangleright A sequence is monotonic iff the sequence is both non-increasing and non-decreasing.

Definition

- A sequence $\{a_n\}$ is increasing iff $a_n < a_{n+1}$.
- A sequence $\{a_n\}$ is non-decreasing iff $a_n \le a_{n+1}$.
- A sequence $\{a_n\}$ is decreasing iff $a_n > a_{n+1}$.
- A sequence $\{a_n\}$ is non-increasing iff $a_n \ge a_{n+1}$.
- \triangleright A sequence is monotonic iff the sequence is both non-increasing and non-decreasing.

Theorem

 \triangleright A non-decreasing sequence converges iff it is bounded above.

Definition

- A sequence $\{a_n\}$ is increasing iff $a_n < a_{n+1}$.
- A sequence $\{a_n\}$ is non-decreasing iff $a_n \le a_{n+1}$.
- A sequence $\{a_n\}$ is decreasing iff $a_n > a_{n+1}$.
- A sequence $\{a_n\}$ is non-increasing iff $a_n \ge a_{n+1}$.
- \triangleright A sequence is monotonic iff the sequence is both non-increasing and non-decreasing.

Theorem

- \triangleright A non-decreasing sequence converges iff it is bounded above.
- \triangleright A non-increasing sequence converges iff it bounded below.

Example

Determine whether the sequence $a_n = \frac{n}{n^2}$ $\frac{n}{n^2+1}$ converges or not.

K ロ ▶ K 레 ▶ K 로 ▶ K 로 ▶ 『로 → 이익단

Example

Determine whether the sequence $a_n = \frac{n}{n^2}$ $\frac{n}{n^2+1}$ converges or not.

Solution: We show that a_n is decreasing.

Example

Determine whether the sequence $a_n = \frac{n}{n^2}$ $\frac{n}{n^2+1}$ converges or not.

Solution: We show that a_n is decreasing. Indeed, the condition

 $a_{n+1} < a_n$

Example

Determine whether the sequence $a_n = \frac{n}{n^2}$ $\frac{n}{n^2+1}$ converges or not.

Solution: We show that a_n is decreasing. Indeed, the condition

$$
a_{n+1} < a_n \quad \Leftrightarrow \quad \frac{n+1}{(n+1)^2+1} < \frac{n}{n^2+1}
$$

Example

Determine whether the sequence $a_n = \frac{n}{n^2}$ $\frac{n}{n^2+1}$ converges or not.

Solution: We show that a_n is decreasing. Indeed, the condition

$$
a_{n+1} < a_n \quad \Leftrightarrow \quad \frac{n+1}{(n+1)^2+1} < \frac{n}{n^2+1}
$$

$$
(n+1)(n^2+1) < n(n^2+2n+2)
$$
Example

Determine whether the sequence $a_n = \frac{n}{n^2}$ $\frac{n}{n^2+1}$ converges or not.

Solution: We show that a_n is decreasing. Indeed, the condition

$$
a_{n+1} < a_n \quad \Leftrightarrow \quad \frac{n+1}{(n+1)^2 + 1} < \frac{n}{n^2 + 1}
$$
\n
$$
(n+1)(n^2 + 1) < n(n^2 + 2n + 2)
$$
\n
$$
n^3 + n^2 + n + 1 < n^3 + 2n^2 + 2n
$$

Example

Determine whether the sequence $a_n = \frac{n}{n^2}$ $\frac{n}{n^2+1}$ converges or not.

Solution: We show that a_n is decreasing. Indeed, the condition

$$
a_{n+1} < a_n \quad \Leftrightarrow \quad \frac{n+1}{(n+1)^2 + 1} < \frac{n}{n^2 + 1}
$$
\n
$$
(n+1)(n^2 + 1) < n(n^2 + 2n + 2)
$$
\n
$$
n^3 + n^2 + n + 1 < n^3 + 2n^2 + 2n
$$

Since $1 < (n^2 + n)$ is true for $n \geq 1$,

Example

Determine whether the sequence $a_n = \frac{n}{n^2}$ $\frac{n}{n^2+1}$ converges or not.

Solution: We show that a_n is decreasing. Indeed, the condition

$$
a_{n+1} < a_n \quad \Leftrightarrow \quad \frac{n+1}{(n+1)^2 + 1} < \frac{n}{n^2 + 1}
$$
\n
$$
(n+1)(n^2 + 1) < n(n^2 + 2n + 2)
$$
\n
$$
n^3 + n^2 + n + 1 < n^3 + 2n^2 + 2n
$$

Since $1 < (n^2 + n)$ is true for $n \geqslant 1$, then $a_{n+1} < a_n$; decreasing.

Example

Determine whether the sequence $a_n = \frac{n}{n^2}$ $\frac{n}{n^2+1}$ converges or not.

Solution: We show that a_n is decreasing. Indeed, the condition

$$
a_{n+1} < a_n \quad \Leftrightarrow \quad \frac{n+1}{(n+1)^2 + 1} < \frac{n}{n^2 + 1}
$$
\n
$$
(n+1)(n^2 + 1) < n(n^2 + 2n + 2)
$$
\n
$$
n^3 + n^2 + n + 1 < n^3 + 2n^2 + 2n
$$

Since $1 < (n^2 + n)$ is true for $n \geqslant 1$, then $a_{n+1} < a_n$; decreasing. The sequence satisfies that $0 < a_n$, bounded below.

Example

Determine whether the sequence $a_n = \frac{n}{n^2}$ $\frac{n}{n^2+1}$ converges or not.

Solution: We show that a_n is decreasing. Indeed, the condition

$$
a_{n+1} < a_n \quad \Leftrightarrow \quad \frac{n+1}{(n+1)^2 + 1} < \frac{n}{n^2 + 1}
$$
\n
$$
(n+1)(n^2 + 1) < n(n^2 + 2n + 2)
$$
\n
$$
n^3 + n^2 + n + 1 < n^3 + 2n^2 + 2n
$$

Since $1 < (n^2 + n)$ is true for $n \geqslant 1$, then $a_{n+1} < a_n$; decreasing. The sequence satisfies that $0 < a_n$, bounded below. We conclude that a_n converges. \triangleleft

The integral test (Sect. 10.3)

 \blacktriangleright Review: Bounded and monotonic sequences.

KORK EX KEY KEY KORA

- \blacktriangleright Application: The harmonic series.
- \blacktriangleright Testing with an integral.
- \blacktriangleright Error estimation in the integral test.

Remarks:

 \blacktriangleright The partial sums of the harmonic series, $s_n = \sum_{n=1}^n$ 1 $\frac{1}{k}$,

 $k=1$

KID KA KERKER E VOOR

Remarks:

 \blacktriangleright The partial sums of the harmonic series, $s_n = \sum_{n=1}^n$

 $k=1$

1 $\frac{1}{k}$,

define an increasing sequence:

Remarks:

 \blacktriangleright The partial sums of the harmonic series, $s_n = \sum_{n=1}^n$ 1 $\frac{1}{k}$,

 $k=1$

define an increasing sequence: $s_{n+1} > s_n$.

Remarks:

 \blacktriangleright The partial sums of the harmonic series, $s_n = \sum_{n=1}^n$ 1 $\frac{1}{k}$,

 $k=1$

K ロ ▶ K @ ▶ K 할 X K 할 X → 할 X → 9 Q Q ^

define an increasing sequence: $s_{n+1} > s_n$.

 \triangleright We now show that $\{s_n\}$ is unbounded from above.

Example

Show that the harmonic series $\sum_{n=0}^{\infty}$ $n=1$ 1 – diverges.
n

Remarks:

 \blacktriangleright The partial sums of the harmonic series, $s_n = \sum_{n=1}^n$ 1 $\frac{1}{k}$,

 $k=1$

define an increasing sequence: $s_{n+1} > s_n$.

 \triangleright We now show that $\{s_n\}$ is unbounded from above.

Example

Show that the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ $n=1$ – diverges.
n

Solution: Notice the following inequalities:

$$
\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \left[\frac{1}{3} + \frac{1}{4}\right] + \left[\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right] + \cdots
$$

Remarks:

 \blacktriangleright The partial sums of the harmonic series, $s_n = \sum_{n=1}^n$ 1 $\frac{1}{k}$,

 $k=1$

define an increasing sequence: $s_{n+1} > s_n$.

 \triangleright We now show that $\{s_n\}$ is unbounded from above.

Example

Show that the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ $n=1$ – diverges.
n

Solution: Notice the following inequalities:

$$
\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \left[\frac{1}{3} + \frac{1}{4}\right] + \left[\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right] + \cdots
$$

$$
\sum_{n=1}^{\infty} \frac{1}{n} > 1 + \frac{1}{2} + \left[\frac{2}{4}\right] + \left[\frac{4}{8}\right] + \cdots
$$

Remarks:

 \blacktriangleright The partial sums of the harmonic series, $s_n = \sum_{n=1}^n$ 1 $\frac{1}{k}$,

define an increasing sequence: $s_{n+1} > s_n$.

 \triangleright We now show that $\{s_n\}$ is unbounded from above.

Example

Show that the harmonic series $\sum_{n=0}^{\infty}$ $n=1$ 1 – diverges.
n

Solution: Notice the following inequalities:

$$
\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \left[\frac{1}{3} + \frac{1}{4}\right] + \left[\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right] + \cdots
$$

$$
\sum_{n=1}^{\infty} \frac{1}{n} > 1 + \frac{1}{2} + \left[\frac{2}{4}\right] + \left[\frac{4}{8}\right] + \cdots \implies \sum_{n=1}^{\infty} \frac{1}{n} \text{ diverges.}
$$

 $k=1$

The integral test (Sect. 10.3)

 \blacktriangleright Review: Bounded and monotonic sequences.

KORK EX KEY KEY KORA

- \blacktriangleright Application: The harmonic series.
- \triangleright Testing with an integral.
- \blacktriangleright Error estimation in the integral test.

Remark:

▶ The idea used above to show that \sum^{∞} $n=1$ 1 – diverges cannot be
n

K □ ▶ K @ ▶ K 할 X K 할 X (할) 10 Q Q Q

generalized to other series.

Remark:

► The idea used above to show that $\sum_{n=1}^{\infty} \frac{1}{n}$ $n=1$ – diverges cannot be
n generalized to other series.

 \triangleright Now we introduce an idea to test the convergence of series.

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

Remark:

- ▶ The idea used above to show that \sum^{∞} $n=1$ 1 – diverges cannot be
n generalized to other series.
- \triangleright Now we introduce an idea to test the convergence of series. The idea is based on calculus.

Remark:

► The idea used above to show that $\sum_{n=1}^{\infty} \frac{1}{n}$ $n=1$ – diverges cannot be
n generalized to other series.

 \triangleright Now we introduce an idea to test the convergence of series. The idea is based on calculus.

Theorem

If $f : [1, \infty) \to \mathbb{R}$ is a continuous, positive, decreasing function, and $a_n = f(n)$, then the following holds:

$$
\sum_{n=1}^{\infty} a_n
$$
 converges \Leftrightarrow $\int_{1}^{\infty} f(x) dx$ converges.

K ロ > K 레 > K 할 > K 할 > 1 를 > 1 이익어

Proof: Recall: $a_n = f(n)$. The proof is based in the pictures:

 4 ロ) 4 \overline{r}) 4 \overline{z}) 4 \overline{z})

唾

Proof: Recall: $a_n = f(n)$. The proof is based in the pictures:

 4 ロ) 4 \overline{r}) 4 \overline{z}) 4 \overline{z})

 \Rightarrow

Proof: Recall: $a_n = f(n)$. The proof is based in the pictures:

モニ マ イ ラ マ ス ラ マ ラ メ コ メ

Proof: Recall: $a_n = f(n)$. The proof is based in the pictures:

 $\mathbf{A} \equiv \mathbf{A} + \mathbf{A} + \mathbf{B} + \mathbf{A} + \math$

Proof: Recall: $a_n = f(n)$. The proof is based in the pictures:

モニット イランド・ミンド (量)

Proof: Recall: $a_n = f(n)$. The proof is based in the pictures:

KORK ERKER ER AGA

Example

Use the integral test to show that \sum^{∞} $n=1$ 1 $\frac{1}{n}$ diverges.

K □ ▶ K @ ▶ K 할 X K 할 X (할) 10 Q Q Q

Example

Use the integral test to show that \sum^{∞} $n=1$ 1 $\frac{1}{n}$ diverges.

Solution: The convergence of the harmonic series $\sum_{n=0}^{\infty}$ $n=1$ 1 $\frac{1}{n}$ is related to the convergence of the integral \int^∞ 1 dx $\frac{1}{x}$.

Example

Use the integral test to show that $\sum_{n=1}^{\infty} \frac{1}{n}$ $n=1$ $\frac{1}{n}$ diverges.

Solution: The convergence of the harmonic series $\sum_{n=0}^{\infty} \frac{1}{n}$ $n=1$ $\frac{1}{n}$ is related to the convergence of the integral \int^∞ 1 dx $\frac{m}{x}$. Since

$$
\ln(n+1) = \int_1^{n+1} \frac{dx}{x}
$$

Example

Use the integral test to show that $\sum_{n=1}^{\infty} \frac{1}{n}$ $n=1$ $\frac{1}{n}$ diverges.

Solution: The convergence of the harmonic series $\sum_{n=0}^{\infty} \frac{1}{n}$ $n=1$ $\frac{1}{n}$ is related to the convergence of the integral \int^∞ 1 dx $\frac{m}{x}$. Since

$$
\ln(n+1) = \int_1^{n+1} \frac{dx}{x} \leqslant \sum_{k=1}^n a_n
$$

Example

Use the integral test to show that $\sum_{n=1}^{\infty} \frac{1}{n}$ $n=1$ $\frac{1}{n}$ diverges.

Solution: The convergence of the harmonic series $\sum_{n=0}^{\infty} \frac{1}{n}$ $n=1$ $\frac{1}{n}$ is related to the convergence of the integral \int^∞ 1 dx $\frac{m}{x}$. Since

$$
\ln(n+1) = \int_1^{n+1} \frac{dx}{x} \leqslant \sum_{k=1}^n a_n \quad \text{and} \quad \ln(n+1) \to \infty
$$

Example

Use the integral test to show that $\sum_{n=1}^{\infty} \frac{1}{n}$ $n=1$ $\frac{1}{n}$ diverges.

Solution: The convergence of the harmonic series $\sum_{n=0}^{\infty} \frac{1}{n}$ $n=1$ $\frac{1}{n}$ is related to the convergence of the integral \int^∞ 1 dx $\frac{m}{x}$. Since

$$
\ln(n+1) = \int_1^{n+1} \frac{dx}{x} \leqslant \sum_{k=1}^n a_n \quad \text{and} \quad \ln(n+1) \to \infty
$$

then the harmonic series $\sum_{n=1}^{\infty}$

$$
\sum_{n=1}^{\infty} \frac{1}{n}
$$
 diverges.

Example

Show whether the series $\sum_{n=0}^{\infty}$ $n=1$ 1 $\frac{1}{1+n^2}$ converges or not.

K □ ▶ K @ ▶ K 할 X K 할 X (할) 10 Q Q Q

Example

Show whether the series
$$
\sum_{n=1}^{\infty} \frac{1}{1+n^2}
$$
 converges or not.

Solution: The convergence of the series $\sum_{n=0}^{\infty}$ $n=1$ 1 $\frac{1}{1+n^2}$ is related to the convergence of the integral $\,\int^\infty$ 1 dx $\frac{1}{1+x^2}$

K ロ ▶ K @ ▶ K 할 > K 할 > 1 할 > 1 이익어

Example

٠

Show whether the series
$$
\sum_{n=1}^{\infty} \frac{1}{1+n^2}
$$
 converges or not.

Solution: The convergence of the series $\sum_{n=0}^{\infty}$ $n=1$ 1 $\frac{1}{1+n^2}$ is related to the convergence of the integral $\,\int^\infty$ 1 dx $\frac{d^{2}y}{1 + x^2}$. Since a_n

AD A REAKEN E VAN

$$
\int_1^n \frac{dx}{1+x^2} = \arctan(x)\big|_1^n
$$

Example

Show whether the series
$$
\sum_{n=1}^{\infty} \frac{1}{1+n^2}
$$
 converges or not.

Solution: The convergence of the series $\sum_{n=0}^{\infty}$ $n=1$ 1 $\frac{1}{1+n^2}$ is related to the convergence of the integral $\,\int^\infty$ 1 dx $\frac{d^{2}y}{1 + x^2}$. Since \int_0^n 1 dx $\frac{dx}{1 + x^2}$ = arctan(x) n $\frac{n}{1} = \left(\arctan(n) - \frac{\pi}{4}\right)$ 4 \setminus

AD A REAKEN E VAN

Example

1

Show whether the series
$$
\sum_{n=1}^{\infty} \frac{1}{1+n^2}
$$
 converges or not.

Solution: The convergence of the series $\sum_{n=0}^{\infty}$ $n=1$ 1 $\frac{1}{1+n^2}$ is related to the convergence of the integral $\,\int^\infty$ 1 dx $\frac{d^{2}y}{1 + x^2}$. Since \int_0^n dx $\frac{dx}{1 + x^2}$ = arctan(x) n $\frac{n}{1} = \left(\arctan(n) - \frac{\pi}{4}\right)$ 4 $\Big) \rightarrow \Big(\frac{\pi}{2}\Big)$ $\frac{\pi}{2} - \frac{\pi}{4}$ 4 .
Example

Show whether the series
$$
\sum_{n=1}^{\infty} \frac{1}{1+n^2}
$$
 converges or not.

Solution: The convergence of the series $\sum_{n=0}^{\infty}$ $n=1$ 1 $\frac{1}{1+n^2}$ is related to the convergence of the integral $\,\int^\infty$ 1 dx $\frac{d^{2}y}{1 + x^2}$. Since \int_0^n 1 dx $\frac{dx}{1 + x^2}$ = arctan(x) n $\frac{n}{1} = \left(\arctan(n) - \frac{\pi}{4}\right)$ 4 $\Big) \rightarrow \Big(\frac{\pi}{2}\Big)$ $\frac{\pi}{2} - \frac{\pi}{4}$ 4 . The inequality $\sum_{n=0}^{\infty}$ $r \infty$

The inequality
$$
\sum_{k=1} a_k \leq a_1 + \int_1^{\infty} f(x) dx
$$
 implies

Example

Show whether the series
$$
\sum_{n=1}^{\infty} \frac{1}{1+n^2}
$$
 converges or not.

Solution: The convergence of the series $\sum_{n=0}^{\infty}$ $n=1$ 1 $\frac{1}{1+n^2}$ is related to the convergence of the integral $\,\int^\infty$ 1 dx $\frac{d^{2}y}{1 + x^2}$. Since \int_0^n 1 dx $\frac{dx}{1 + x^2}$ = arctan(x) n $\frac{n}{1} = \left(\arctan(n) - \frac{\pi}{4}\right)$ 4 $\Big) \rightarrow \Big(\frac{\pi}{2}\Big)$ $\frac{\pi}{2} - \frac{\pi}{4}$ 4 . The inequality $\sum_{k=1}^{\infty}a_k\leqslant a_1+\int_{0}^{\infty}b_k$ 1 $f(x)$ dx implies

$$
\sum_{n=1}^{\infty} \frac{1}{1+n^2} \leqslant \frac{1}{2} + \frac{\pi}{4}
$$

Example

Show whether the series
$$
\sum_{n=1}^{\infty} \frac{1}{1+n^2}
$$
 converges or not.

Solution: The convergence of the series $\sum_{n=0}^{\infty}$ $n=1$ 1 $\frac{1}{1+n^2}$ is related to the convergence of the integral $\,\int^\infty$ 1 dx $\frac{d^{2}y}{1 + x^2}$. Since a_n

$$
\int_1^n \frac{dx}{1+x^2} = \arctan(x)\big|_1^n = \left(\arctan(n) - \frac{\pi}{4}\right) \to \left(\frac{\pi}{2} - \frac{\pi}{4}\right).
$$

The inequality $\sum_{k=1}^{\infty}a_k\leqslant a_1+\int_{0}^{\infty}b_k$ $k=1$ 1 $f(x)$ dx implies \sum^{∞} $n=1$ 1 $\frac{1}{1+n^2} \leqslant \frac{1}{2}$ $\frac{1}{2} + \frac{\pi}{4}$ $\frac{\pi}{4}$ \Rightarrow $\sum_{n=1}^{\infty}$ $n=1$ 1 $\frac{1}{1+n^2}$ converges.

 $\mathbf{E} = \mathbf{A} \oplus \mathbf{A} + \mathbf{A} \oplus \mathbf{A} + \mathbf{A} \oplus \mathbf{A} + \mathbf{A} \oplus \mathbf{A}$

Example

Show whether the series $\sum_{n=0}^{\infty}$ $n=1$ $\frac{1}{\sqrt{1-\frac{1}{2}}}$ n √ $n+1$ converges or not.

K □ ▶ K @ ▶ K 할 X K 할 X (할) 10 Q Q Q

Example

Show whether the series
$$
\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\sqrt{n+1}}
$$
 converges or not.

Solution: The convergence of the series
$$
\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\sqrt{n+1}}
$$
 is related to the convergence of the integral
$$
\int_{1}^{\infty} \frac{dx}{\sqrt{x}\sqrt{x+1}}
$$
.

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ | 할 | X 9 Q Q

Example

Show whether the series
$$
\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\sqrt{n+1}}
$$
 converges or not.

Solution: The convergence of the series $\sum_{n=0}^{\infty}$ $n=1$ $\frac{1}{\sqrt{1-\frac{1}{2}}}$ n √ $n+1$ is related to the convergence of the integral \int^∞ 1 $\frac{dx}{\sqrt{2}}$ $\overline{\mathbf{x}}$ √ $x + 1$.

K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ (할) 1000

Limit test for improper integrals:

Example

Show whether the series
$$
\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\sqrt{n+1}}
$$
 converges or not.

Solution: The convergence of the series $\sum_{n=0}^{\infty}$ $n=1$ $\frac{1}{\sqrt{1-\frac{1}{2}}}$ n √ $n+1$ is related to the convergence of the integral \int^∞ 1 $\frac{dx}{\sqrt{2}}$ $\overline{\mathbf{x}}$ √ $x + 1$. Limit test for improper integrals: $\lim_{x\to\infty}$ $\frac{1}{\sqrt{1-\frac{1}{2}}}$ $\overline{\mathsf{x}}$ √ $x + 1$ $=\lim_{x\to\infty}$ 1 $\frac{1}{x}$.

AD A REAKEN E VAN

Example

Show whether the series
$$
\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\sqrt{n+1}}
$$
 converges or not.

Solution: The convergence of the series $\sum_{n=0}^{\infty}$ $n=1$ $\frac{1}{\sqrt{1-\frac{1}{2}}}$ n √ $n+1$ is related to the convergence of the integral \int^∞ 1 $\frac{dx}{\sqrt{2}}$ $\overline{\mathbf{x}}$ √ $x + 1$. $\frac{1}{\sqrt{1-\frac{1}{2}}}$ 1

Limit test for improper integrals: $\lim_{x\to\infty}$ $\overline{\mathsf{x}}$ √ $x + 1$ $=\lim_{x\to\infty}$ $\frac{1}{x}$.

KORK ERKER ER AGA

Since
$$
\int_{1}^{\infty} \frac{dx}{x}
$$
 diverges,

Example

Show whether the series
$$
\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\sqrt{n+1}}
$$
 converges or not.

Solution: The convergence of the series $\sum_{n=0}^{\infty}$ $n=1$ $\frac{1}{\sqrt{1-\frac{1}{2}}}$ n √ $n+1$ is related to the convergence of the integral \int^∞ 1 $\frac{dx}{\sqrt{2}}$ $\overline{\mathbf{x}}$ √ $x + 1$. $\frac{1}{\sqrt{1-\frac{1}{2}}}$ √ 1 $\frac{1}{x}$.

Limit test for improper integrals:
$$
\lim_{x \to \infty} \frac{1}{\sqrt{x}\sqrt{x+1}} = \lim_{x \to \infty}
$$

Since $\int_{1}^{\infty} \frac{dx}{x}$ diverges, then $\int_{1}^{\infty} \frac{dx}{\sqrt{x}\sqrt{x+1}}$ diverges.

Example

Show whether the series
$$
\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\sqrt{n+1}}
$$
 converges or not.

Solution: The convergence of the series $\sum_{n=0}^{\infty}$ $n=1$ $\frac{1}{\sqrt{1-\frac{1}{2}}}$ n √ $n+1$ is related to the convergence of the integral \int^∞ 1 $\frac{dx}{\sqrt{2}}$ $\overline{\mathbf{x}}$ √ $x + 1$.

Limit test for improper integrals: $\lim_{x\to\infty}$ $\frac{1}{\sqrt{1-\frac{1}{2}}}$ $\overline{\mathsf{x}}$ √ $x + 1$ $=\lim_{x\to\infty}$ 1 $\frac{1}{x}$.

Since
$$
\int_{1}^{\infty} \frac{dx}{x}
$$
 diverges, then $\int_{1}^{\infty} \frac{dx}{\sqrt{x}\sqrt{x+1}}$ diverges.

Integral test for series implies

$$
\therefore \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\sqrt{n+1}} \text{ diverges.} \qquad \text{and} \qquad
$$

The integral test (Sect. 10.3)

 \blacktriangleright Review: Bounded and monotonic sequences.

KORK EX KEY KEY KORA

- \blacktriangleright Application: The harmonic series.
- \blacktriangleright Testing with an integral.
- \blacktriangleright Error estimation in the integral test.

Error estimation in the integral test.

Theorem If $f : [1, \infty) \to \mathbb{R}$ is a continuous, positive, decreasing function, and the series $\displaystyle{\sum_{n=1}^{n}a_k=s_n\to S}$, where $a_n=f(n)$, then the remainder $R_n = S - s_n$ satisfies \int^{∞} $n+1$ $f(x) dx \le R_n \le \int_{0}^{\infty}$ n $f(x)$ dx.

AD A REAREA E ARA

Error estimation in the integral test.

Theorem If $f : [1, \infty) \to \mathbb{R}$ is a continuous, positive, decreasing function, and the series $\displaystyle{\sum_{n=1}^{n}a_k=s_n\to S}$, where $a_n=f(n)$, then the remainder $R_n = S - s_n$ satisfies \int^{∞} $n+1$ $f(x) dx \le R_n \le \int_{0}^{\infty}$ n $f(x)$ dx.

モニ マ イ ラ マ ス ラ マ ラ メ コ メ

Error estimation in the integral test.

Theorem If $f : [1, \infty) \to \mathbb{R}$ is a continuous, positive, decreasing function, and the series $\displaystyle{\sum_{n=1}^{n}a_k=s_n\to S}$, where $a_n=f(n)$, then the remainder $R_n = S - s_n$ satisfies \int^{∞} $n+1$ $f(x) dx \le R_n \le \int_{0}^{\infty}$ n $f(x)$ dx.

 4 ロ) 4 \overline{r}) 4 \overline{z}) 4 \overline{z})