Name: ______________________________ **ID Number:** ____________________

TA: ____________________________________ **Section:** ____________________

MTH 235
Practice Final Exam
April 30, 2010
120 minutes
Chptrs: 2, 3, 5, 6, 7, 10.

No notes. No books. No Calculators.
If any question is not clear, ask for clarification.
No credit will be given for illegible solutions.
If you present different answers for the same problem, the worst answer will be graded.
Show all your work. Box your answers.

Signature: ____

Problem	Points	Score
1	15	
2	15	
3	15	
4	15	
5	15	
6	10	
7	10	
8	15	
9	10	
10	10	
11	10	
12	15	
13	15	
14	15	
15	15	
Σ	200	

1. (15 points) Find the most general solution y of the equation

$$y' = \frac{e^x \sin(y) + 2x}{3y - e^x \cos(y)}.$$

2. (15 points) Find the general solution y to the differential equation

$$t^2 y' + 2t\, y = y^3.$$

3. (15 points) Find all solutions y to the equation below and leave them in implicit form,

$$y' = \frac{x^2 + 3xy + y^2}{x^2}.$$

4. (15 points) Find the general solution y to the differential equation

$$2y'' + y' = t + 2\sin(t).$$

5. (15 points) Find the general solution y to the equation

$$y'' + 2y' + y = \frac{e^{-t}}{t}.$$

6. (10 points) Find the general solution y of

$$4x^2\, y'' + 8x\, y' + y = 0, \qquad x > 0.$$

7. (10 points) Find all singular points of the equation below and determine which of those are regular singular points, where

$$x(x-2)^2\, y'' + 3(x+2)\, y' + (x-3)\, y = 0.$$

8. (15 points) Use power series centered at $x_0 = 1$ to look for two linearly independent solutions $y_1(x)$ and $y_2(x)$ of the differential equation

$$-3xy'' + 2y' + y = 0.$$

(a) Find the recurrence relation for the power series coefficients.

(b) Find the first two non-zero terms of the power series for each of the linearly independent solutions y_1 and y_2.

9. (10 points) Find the Laplace transform of the function f given by

$$f(t) = \begin{cases} 3 & \text{if } \ 0 \leqslant t < \pi, \\ \sin(t-\pi) & \text{if } \qquad t \geqslant \pi. \end{cases}$$

10. (10 points) Find an explicit expression (that is, without using convolutions) of the inverse Laplace transform of the function F given by

$$F(s) = \frac{e^{-\pi s}}{(s^2+1)(s^2+4)}.$$

11. (10 points) Given a continuous but otherwise arbitrary function g, use the Laplace transform method to find the solution y to the initial value problem

$$y'' + 4y' + 8y = g(t), \qquad y(0) = y'(0) = 0.$$

Express the solution y in terms of appropriate convolutions with function g.

12. (15 points) Find the general solution $\boldsymbol{x}$ to the equation

$$\boldsymbol{x}' = A\boldsymbol{x}, \qquad A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}.$$

Sketch a phase portrait with few solution trajectories.

13. (15 points) Find the solution $\boldsymbol{x}$ to the initial value problem

$$\boldsymbol{x}' = A\boldsymbol{x}, \qquad \boldsymbol{x}(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \qquad A = \begin{bmatrix} 3 & -18 \\ 2 & -9 \end{bmatrix}.$$

14. (15 points) Find every positive eigenvalue λ and nonzero function y, solutions of the boundary value problem

$$y'' - 4y' + 4y = -\lambda y, \qquad y(0) = 0, \quad y(3) = 0.$$

15. (15 points) Consider the function $f(x) = -1$, defined for $-3 < x < 0$.

(a) Sketch the graph of the odd periodic extension of period $T = 6$ of the function f above. Sketch the graph of this extension for at least three periods.

(b) Determine the Fourier series of this odd periodic extension of f.

You are allowed to use the Laplace transform table on page 317 in the textbook.

Nevertheless, this is a short list of Laplace transforms and Laplace transform properties that could be useful for the exam. We use the notation $\mathcal{L}[f(t)] = F(s)$.

$$
\begin{aligned}
&f(t) = e^{at} && F(s) = \frac{1}{s-a} && s > \max\{a, 0\},\\
&f(t) = t^n && F(s) = \frac{n!}{s^{(n+1)}} && s > 0,\\
&f(t) = \sin(at) && F(s) = \frac{a}{s^2+a^2} && s > 0,\\
&f(t) = \cos(at) && F(s) = \frac{s}{s^2+a^2} && s > 0,\\
&f(t) = \sinh(at) && F(s) = \frac{a}{s^2-a^2} && s > 0,\\
&f(t) = \cosh(at) && F(s) = \frac{s}{s^2-a^2} && s > 0,\\
&f(t) = t^n e^{at} && F(s) = \frac{n!}{(s-a)^{(n+1)}} && s > \max\{a, 0\},\\
&f(t) = e^{at}\sin(bt) && F(s) = \frac{b}{(s-a)^2+b^2} && s > \max\{a, 0\},\\
&f(t) = e^{at}\cos(bt) && F(s) = \frac{s-a}{(s-a)^2+b^2} && s > \max\{a, 0\}.
\end{aligned}
$$

The following Laplace transforms could also be useful, where u denotes the step function at $t = 0$, and δ the Dirac delta generalized function:

$$
\mathcal{L}[u(t-c)] = \frac{e^{-cs}}{s}, \qquad \mathcal{L}[\delta(t-c)] = e^{-cs}.
$$

The following relations could also be useful:

$$
\begin{aligned}
e^{-cs}\,\mathcal{L}\big[f(t)\big] &= \mathcal{L}\big[u(t-c)\,f(t-c)\big],\\
\mathcal{L}\big[e^{ct}\,f(t)\big] &= F(s-c),\\
\mathcal{L}\big[f^{(n)}(t)\big] &= s^n F(s) - s^{(n-1)} f(0) - \cdots - f^{(n-1)}(0).
\end{aligned}
$$