**MATH 234** **FINAL EXAM** **April 30, 2001**

**Name(print)_____ Student Number_____**
**Section Number_____**

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|---|---|---|---|---|---|---|---|
| **Points** | | | | | | | |

**Instructions:**
1.Since grading will be based on method you must show all work .
2.Boldfaced letters indicate vecturs such as **F** or **k.**
3.Check that your exam has the 12 problems.

1.(16 pts) Let A=(1,2,3), B=(6,5,4) and C= (8,9,7).

a) Find $\vec{AB}$ *and* $\vec{AC}$

b) Find $\vec{AB}+\vec{AC}$

c) Find $\vec{AB}\bullet\vec{AC}$

d) Find $\vec{AB}\times\vec{AC}$

2.(16 pts) a) Find the parametric equations for the line through the point (1,0,–1) and perpendicular to the plane $2x-3y+5z=35$.

b) Find the point where this line intersects the plane.

3.(18 pts) The velocity of a particle is given by $\vec{v}(t) = t^2\,\vec{i} + (t^3+1)\,\vec{j}$ and the particle is at the point (2,1) when t=0.

a) Where is the particle when t=2?

b) Write the integral (DO NOT EVALUATE) that gives the arc-length the particle travels when $0 \le t \le 2$.

c) Find the acceleration of the particle.

4.(20 pts) a) Draw a rough sketch of the surface $z = 2x^2 + 3y^2 + 5$.

b) Find the equation of the tangent plane to the surface at the point (1,1,10).

5.(15pts) Let w=f(x,y) and $x = s^2 + t^2, y = st^2$. If $\frac{\partial f}{\partial x} = x - y$ *and* $\frac{\partial f}{\partial y} = y - x$ find $\frac{\partial w}{\partial s}$ *and* $\frac{\partial w}{\partial t}$ in terms of s and t.

6.(15 pts) Find all critical points of the function $f(x, y) = 2x^2 + 8xy + y^4$ and determine whether they are a local maximum, a local minimum or a saddle point.

7.(16 pts) Given the integral $\int_0^1 \int_{\sqrt[3]{x}}^1 e^{\frac{x}{y}}\, dydx$

a) Sketch the region of integration.

b) Evaluate the integral by reversing the order of integration..

8, (16pts) Consider the force field $\vec{F} = yz\,\vec{i} + xz\,\vec{j} - xy\vec{k}$

a) Set up a line integral for the work done by this force field in moving a particle along the curve $\vec{r}(t) = t^3\,\vec{i} + t^2\,\vec{j} + t\,\vec{k}, 0 \le t \le 2.$

b) Evaluate this integral.

9. (22 pts) Let $\vec{F} = (y\cos z - yze^x)\,\vec{i} + (x\cos z - ze^x)\,\vec{j} - (xy\sin z + ye^x)\,\vec{k}$

a) Show that this force field is conservative.

b) Find a potential function for this vector field.

c) Evaluate $\int_C \vec{F} \bullet \vec{dr}$ where C is the curve $\vec{r}(t) = t\,\vec{i} + t^2\,\vec{j} + pt^3\,\vec{k},\ 0 \le t \le 1$

10. (14 pts) Use Green's Theorem to evaluate the integral $\int_C M(x,y)dx + N(x,y)dy$ *where* $M(x,y) = y + e^x$ *and* $N(x,y) = 2x^2 + \cos y$ and C is the triangle with vertices (0,0), (0,2) and (1,1) traversed counterclockwise.

11. (18 pts) Find the surface area of that portion of the paraboloid $z = 4 - x^2 - y^2$ that lies above the plane z=0. Use polar coordinates to evaluate the integral.

12. (14 pts) Use Stokes's Theorem to evaluate $\iint_S \vec{\nabla} \times (y\vec{i}) \bullet \vec{n}\, ds$ where S is the hemisphere

$x^2 + y^2 + z^2 = 1, z \geq 0$ *and* $\vec{n}$ is the outward unit normal to S.