MATH 234 **FINAL EXAM** **December 11, 2001**

Name(print)__________________________ Student Number_________
Section Number_________

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Points							

Instructions:
1.Since grading will be based on method you must <u>show all work .</u>
2.Boldfaced or lined letters indicate vectors such as **F** or $\vec{k}$ **.**
3.Check that your exam has the 12 problems.
4. <u>No calculators or formula sheets</u> are allowed.

1. (16 pts.) Let **A** = –2**i** + 2**j** + **k** and **B** = 2**i** + 6**j** – 2**k**

a) **– A** + 2**B = ________________________**

b) **A** = ___________

c) **A** **B** = ____________

d) **A** **B =** __________________

2. (18 pts.) A particle moves along a curve with velocity vector
$\vec{v} = \vec{i} + \sqrt{2}\ t\vec{j} + t^2\vec{k}, 1 \le t \le 2$

a) At t = 1 the particle is at the point $(0, \frac{\sqrt{2}}{2}, \frac{2}{3})$. Find $\vec{r}(t)$ *and* $\vec{r}(2)$.

b) How far did the particle travel in going between these two points (that is, from t = 1 to t = 2)?

c) Find the acceleration vector of the particle.

3. (16 points) Let $f(x, y, z) = z + x^2 + 2y^2$

a) Sketch the part of the level surface f(x,y,z) = 4 that lies above the plane z = 0 .

b) Write the equation of the tangent plane to this level surface at the point (–1,1,1)

4. (14 pts.) If w = f(x,y) is a differentiable function of x and y and if $x = r\cos\theta$ *and* $y = r\sin\theta$, *find* $\dfrac{\partial w}{\partial \theta}$ in terms of $\dfrac{\partial f}{\partial x}$ *and* $\dfrac{\partial f}{\partial y}$ (since you do not know the function f(x,y), f_x *and* f_y should appear in your answer).

5. (18 pts.) a) $z^3 + xyz - 2 = 0$ implicitly defines z as a function of x and y. Find $\dfrac{\partial z}{\partial x}$ at the point (1,1,1).

   c) If $f(x,y) = ye^{xy}$ *find* f_{yx}

6. (18 pts.) Find all the critical points of $f(x,y) = x^3 + 6xy + 3y^2 - 9x$ and classify each critical point (clearly giving your reason) as a local maximum, local minimum or saddle point.

7. (18 pts.) Let $I = \int_1^e \int_0^{\ln x} y\,dy\,dx$

a) Sketch the region of integration (e is approximately 2.7).

b) Write I with the order of integration reversed.

c) Evaluate one of the integrals.

8. (16 pts.) Write an integral which gives the surface area of the surface cut from the hemisphere $x^2 + y^2 + z^2 = 6, z \geq 0$ by the cylinder $(x-1)^2 + y^2 = 1$. Your final answer should be written in cylindrical coordinates. **DO NOT EVALUATE** the integral

9. (18 pts.) Let the curve C be given by

$\vec{r}(t) = (\cos t + t\sin t)\vec{i} + (\sin t - t\cos t)\vec{j},\ 0 \le t \le 2$

a) Evaluate $\int_C (x^2 + y^2)\,ds$

b) Evaluate $\int_C \vec{F} \bullet d\vec{r}$ *where* $\vec{F} = x\vec{i} + y\vec{j}$

10. (16 pts.) Use a double integral to evaluate $\oint_C (2x - 6y)\,dx + (y - 4x)\,dy$ where C is the cardioid $r = 1 + \cos q$ with counter-clockwise rotation.

11. (16 pts.) a) Find a potential function for $\vec{F} = (y^3 + z^3)\vec{i} + (3xy^2 + 1)\vec{j} + 3xz^2\vec{k}$

b) Evaluate $\int_C (y^3 + z^3)dx + (3xy^2 + 1)dy + 3xz^2 dz$ where the path C runs from (1,0,1) to (2,2,2) along the two line segments from (1,0,1) to (1,1,0) and from (1,1,0) to (2,2,2).

12. (16 pts) R is the solid region in the first octant that lies beneath the plane 2x + 3y + 2z = 6. Let S be the boundary of R (S consists of 4 triangles). If $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ use the Divergence Theorem to write $\iint_S \vec{F} \bullet \vec{n}\, ds$ as a triple integral. **DO NOT EVALUATE THE INTEGRAL.**