# MATH 234 EXAM#1

Name ...... Solutions ......

Work all 6 problems. Show ALL your work. No calculators may be used on this test.
Total : 50 points.

1. (10 points) Let $\mathbf{v} = (4, 1, 3)$ and $\mathbf{w} = (2, -1, 4)$ be vectors in $\mathbf{R}^3$ and let $P\,(1, 2, 3)$ be a point.

a) Compute $\mathbf{v} \cdot \mathbf{w}$

$$\vec{v} \cdot \vec{w} = 8 + (-1) + 12 = 19$$

b) Compute $\mathbf{v} \times \mathbf{w}$

$$\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 1 & 3 \\ 2 & -1 & 4 \end{vmatrix} = \vec{i}\,(4+3) - \vec{j}\,(16-6) + \vec{k}\,(-4-2)$$

$$= 7\vec{i} - 10\vec{j} - 6\vec{k}$$

c) Find the projection, $\text{proj}_{\mathbf{v}}\mathbf{w}$, of $\mathbf{w}$ onto $\mathbf{v}$.

$$\text{proj}_{\vec{v}}\,\vec{w} = \left(\vec{w} \cdot \vec{v} / |\vec{v}|^2\right) \vec{v} = \left(19/26\right)(4\vec{i} + \vec{j} + 3\vec{k})$$

$$|\vec{v}|^2 = 16 + 1 + 9 = 26$$

d) Write down parametric equations for the line through $P$ in the direction of $\mathbf{v}$.

$$x = 1 + 4t, \quad y = 2 + t, \quad z = 3 + 3t$$

e) Find an equation for the plane through $P$ with normal vector $\mathbf{w}$.

$$0 = 2(x-1) - 1(y-2) + 4(z-3)$$

2. (7 points) The lines $L_1$ defined by $X(t) = (-1+t, 2+t, 1-t)$, and $L_2$ defined by $Y(s) = (1-4s, 1+2s, 2-2s)$ intersect in a point. Find the point. Find an equation for the plane containing $L_1$ and $L_2$.

pt of intersection satisfies:

1) $-1+t = 1-4s$

2) $2+t = 1+2s$

3) $1-t = 2-2s$

1) $\Rightarrow$ $t = 2-4s$

2) then gives $2+2-4s = 1+2s$

$3 = 6s$

$s = \frac{1}{2}$ so $t = 2 - \frac{4}{2} = 0$

3) says $1 = 2-1$

$\therefore$ pt of intersection is $(-1, 2, 1)$

plane contains the directions:

$\vec{v} = (1, 1, -1)$ $\quad$ $\vec{w} = (-4, 2, -2)$

Hence the normal $\vec{n}$ is

$$\vec{n} = \vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ -4 & 2 & -2 \end{vmatrix}$$

$$= \vec{i}(-2+2) - \vec{j}(-2-4) + \vec{k}(2+4)$$

$$= 6\vec{j} + 6\vec{k}$$

equation:

$$0 = 6(y-2) + 6(z-1)$$

3. (7 points) Find the point in which the line through the origin perpendicular to the plane $3x - 4y + z = 6$ meets the plane $2x - y \cdot \ -3z = 7$.

line through origin perpendicular to the plane is:

$$x = 3t, \quad y = -4t, \quad z = t$$

intersects the plane $2x - y - 3z = 7$

$$2(3t) - (-4t) - 3t = 7$$

$$6t + 4t - 3t = 7$$

$$7t = 7$$

$$t = 1$$

point of intersection: $(3, -4, 1)$

4. (7 points) Find a plane through the points $P(1,2,3)$ and $Q(3,2,1)$ that is perpendicular to the plane $4x - y + 2z = 7$.

The direction $\vec{PQ} = (2, 0, -2)$ lies on the required plane. Since the required plane is perpendicular to $4x - y + 2z = 7$ the direction $\vec{v} = (4, -1, 2)$ lies on the required plane. Therefore the required normal $\vec{n}$ is:

$$\vec{n} = \vec{PQ} \times \vec{v}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & -2 \\ 4 & -1 & 2 \end{vmatrix}$$

$$= \vec{i}(-2) - \vec{j}(4+8) + \vec{k}(-1)$$

$$= -2\vec{i} - 12\vec{j} - \vec{k}$$

equation of plane:

$$0 = -2(x-1) - 12(y-2) - 1(z-3).$$

5. (7 points) Find the distance between the planes $2x+3y-z=6$ and $2x+3y-z=12$.

The planes are parallel. The distance between the planes is the distance of any point on one plane, say $2x+3y-z=6$, to the other plane.

Find a pt on $2x+3y-z=6$:

Let $S=(1,1,-1)$. Let $\mathcal{P}=\{2x+3y-z=12\}$

Then $\mathrm{dist}(S,\mathcal{P})$ is:

1) find a pt on $\mathcal{P}$:

$P=(2,3,1)$

$\vec{PS}=(-1,-2,-2)$

2) $\vec{n}$ normal to $\mathcal{P}$ $\quad \vec{n}=(2,3,-1)$ $\quad |\vec{n}|=(4+9+1)^{\frac{1}{2}}$

$=\sqrt{14}$

3) $\mathrm{dist}=\left|\vec{PS}\cdot \vec{n}/|\vec{n}|\right|$

$=\left|\frac{1}{\sqrt{14}}(-2-6+2)\right|$

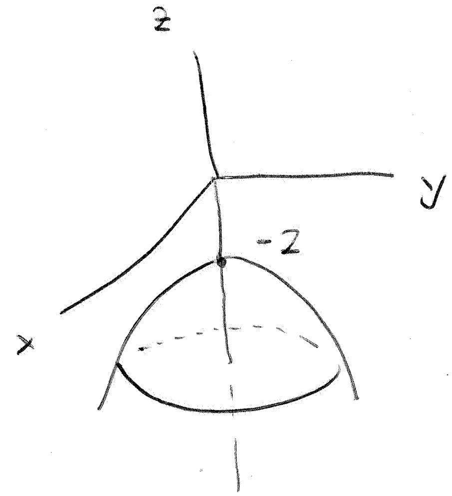
$=6/\sqrt{14}$

6. (12 points) Identify and sketch the surfaces:

a) $z = -x^2 - 2y^2 - 2$

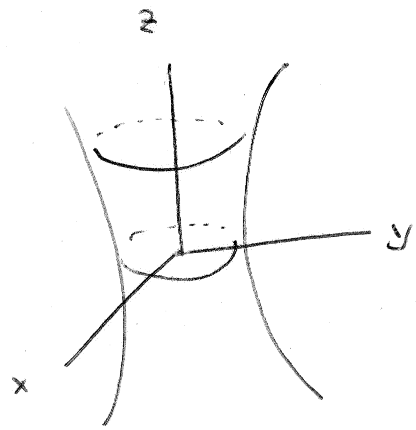
$-(z+2) = x^2 + 2y^2$

elliptical paraboloid



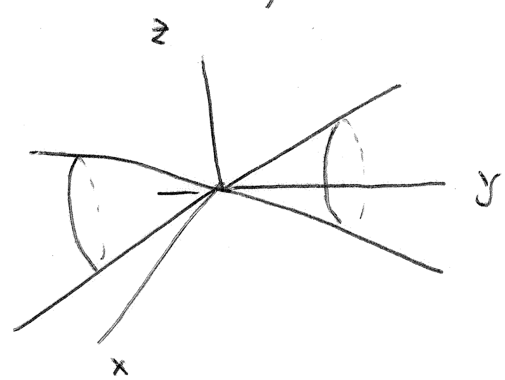

b) $y^2 + 3x^2 - 4z^2 = 1$

one-sheeted hyperboloid




c) $9x^2 + 6z^2 = 2y^2$

elliptical cone




d) $y = x^3 + 2$

generalized cylinder