

**MTH 234**  
**Michigan State University**  
**Department of Mathematics**

Name: \_\_\_\_\_

PID: \_\_\_\_\_

Section No: \_\_\_\_\_

<b>Problem</b>	<b>Total</b>	<b>Score</b>
1	16	
2	16	
3	17	
4	17	
5	16	
6	17	
7	17	
8	17	
9	17	
10	16	
11	17	
12	17	
<b>Total</b>	200	

Michigan State University  
Department of Mathematics

Name: \_\_\_\_\_ PID: \_\_\_\_\_ Section No: \_\_\_\_\_

Signature: \_\_\_\_\_

Total Score:

1. Check that you have pages 1 through 16 and that none are blank.
2. Fill in the information at the top of the page.
3. You will need a pen or pencil and this booklet for the exam. Please clear everything else from your desk.
4. The use of calculators, cell phones, or any other electronic device as an aid to writing this exam is strictly prohibited.
5. The grading of this exam is based on your method. **Show all of your work.** (There are problems however that will be graded right or wrong.) If you need additional space, use the backs of the exam pages.
6. If you present different answers, the worst answer will be graded.
7. 

<b>Box your answers.</b>
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**1.** (16 points)

- (a) Find a unit vector in the opposite direction of  $\mathbf{v} = \langle 1, 2, 3 \rangle$ .
- (b) Find the scalar projection of  $\mathbf{w} = \langle 1, -1, 2 \rangle$  onto  $\mathbf{v}$ .
- (c) Find the vector projection of  $\mathbf{w}$  onto  $\mathbf{v}$ .

**2.** (16 points) Find the equation of the plane that contains the lines  $\mathbf{r}_1(t) = \langle 1, 2, 3 \rangle t$  and  $\mathbf{r}_2(t) = \langle 1, 1, 0 \rangle + \langle 1, 2, 3 \rangle t$ .

**3.** (17 points) A particle moves along the curve  $\mathbf{r}(t) = \langle \sin(2t^2), t^3, \cos(2t^2) \rangle$ , for  $t \geq 0$ .

- (a) Find the velocity  $\mathbf{v}(t)$  and acceleration  $\mathbf{a}(t)$  functions of the particle.
- (b) Find the arc length function for the curve  $\mathbf{r}(t)$  measured from the point where  $t = 0$ , in the direction of increasing  $t$ .

**4.** (17 points)

- (a) Find and sketch the domain of the function  $f(x, t) = \ln(t - x^2)$ .
- (b) Determine whether the function  $f$  above is solution of the wave equation  $f_{tt} - f_{xx} = 0$ .

**5.** (16 points)

- (a) Find the tangent plane approximation of  $f(x, y) = \sin(2x + 5y)$  at the point  $(-5, 2)$ .
- (b) Use the linear approximation computed above to approximate the value of  $f(-4.8, 2.1)$ .

**6.** (17 points) Find the absolute maximum and absolute minimum of the function  $f(x, y) = x^2 + 3y^2 - 2xy$  in the triangle formed by the lines  $y = 0$ ,  $x = 1$  and  $y = x$ .

**7.** (17 points)

(a) Sketch the region of integration,  $D$ , whose area is given by the double integral

$$\int \int_D dA = \int_0^2 \int_{\frac{3}{2}x}^{3\sqrt{x/2}} dy dx.$$

(b) Compute the double integral given in (a).

(c) Change the order of integration in the integral given in (a).

**8.** (17 points) Transform to polar coordinates and then evaluate the integral

$$I = \int_{-1}^1 \int_{-\sqrt{1-y^2}}^0 \ln(x^2 + y^2 + 1) dx dy.$$

**9.** (17 points) Find the component  $\bar{z}$  of the centroid for a wire lying along the curve given by  $\mathbf{r}(t) = \langle t \cos(t), t \sin(t), (2\sqrt{2}/3)t^{3/2} \rangle$ , for  $t \in [0, 1]$ .

**10.** (16 points) Use the Green Theorem area formula to find the area of the region enclosed by the curve  $\mathbf{r}(t) = \langle \cos^2(t), \sin^2(t) \rangle$  for  $t \in [0, \pi/2]$ . (16.4.23).

**11.** (17 points) Find the flux of  $\nabla \times \mathbf{F}$  outward through the surface  $S$ , where  $\mathbf{F} = \langle -y, x, x^2 \rangle$  and  $S = \{x^2 + y^2 = a^2, z \in [0, h]\} \cup \{x^2 + y^2 \leq a^2, z = h\}$ .

**12.** (17 points) Find the outward flux of the field  $\mathbf{F} = \langle x^2, -2xy, 3xz \rangle$  across the boundary of the region  $D = \{x^2 + y^2 + z^2 \leq 4, x \geq 0, y \geq 0, z \geq 0\}$ .