

Name: \_\_\_\_\_ ID Number: \_\_\_\_\_

TA: \_\_\_\_\_ Section Time: \_\_\_\_\_

Math 20D

Exam 2.

May 19, 2008

*No calculators or any other devices are allowed on this exam.*

*Read each question carefully. If any question is not clear, ask for clarification.*

*Write your solutions clearly and legibly; no credit will be given for illegible solutions.*

*Answer each question completely, and show all your work.*

1. (a) (10 points) Find the general solution  $y(t)$  to the differential equation

$$9y'' - 12y' + 4y = 0. \quad (1)$$

- (b) (10 points) Find the unique solution to the initial value problem given by Eq. (1) and satisfying the initial conditions

$$y(0) = -2, \quad y'(0) = -\frac{1}{3}.$$

#	Score
1	
2	
3	
4	
$\Sigma$	

2. (a) (15 points) Use the method of under-determined coefficients to find the general solution  $y(t)$  to the differential equation

$$y'' - 2y' - 3y = 3te^{2t}. \quad (2)$$

- (b) (10 points) Find the unique solution to the initial value problem given by Eq. (2) and satisfying the initial conditions

$$y(0) = \frac{1}{3}, \quad y'(0) = -\frac{22}{3}.$$

3. (20 points) Decide whether the set of vectors shown below is linearly dependent or independent. In the case that the set of vectors is linearly dependent, express one of them as a linear combination of the other two.

$$\left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ 7 \\ 10 \end{bmatrix} \right\}$$

4. (a) (15 points) Find a set of fundamental solutions to the equation

$$\mathbf{x}'(t) = \begin{bmatrix} -1 & 1 \\ 4 & 2 \end{bmatrix} \mathbf{x}(t). \quad (3)$$

- (b) (10 points) Graph in a phase diagram the trajectories of the fundamental solutions found in part (4a). Furthermore, do a *qualitative* sketch of the trajectories of several linear combinations of these fundamental solutions.
- (c) (10 points) Find the solution to the initial value problem given by Eq. (3) and the initial condition

$$\mathbf{x}(0) = \begin{bmatrix} -1 \\ 21 \end{bmatrix}$$