

Name: _____ ID Number: _____

TA: _____ Section Time: _____

Math 20D.
Exam 2
May 19, 2008

No calculators or any other devices are allowed on this exam.

Read each question carefully. If any question is not clear, ask for clarification.

Write your solutions clearly and legibly; no credit will be given for illegible solutions.

Answer each question completely, and show all your work.

1. (a) (10 points) Find the general solution $y(t)$ to the differential equation

$$9y'' - 12y' + 4y = 0. \quad (1)$$

- (b) (10 points) Find the unique solution to the initial value problem given by Eq. (1) and satisfying the initial conditions

$$y(0) = 2, \quad y'(0) = \frac{1}{3}.$$

#	Score
1	
2	
3	
4	
Σ	

2. (a) (15 points) Use the method of under-determined coefficients to find the general solution $y(t)$ to the differential equation

$$y'' - 2y' - 3y = 3te^{2t}. \quad (2)$$

- (b) (10 points) Find the unique solution to the initial value problem given by Eq. (2) and satisfying the initial conditions

$$y(0) = \frac{4}{3}, \quad y'(0) = \frac{23}{3}.$$

3. (20 points) Decide whether the set of vectors shown below is linearly dependent or independent. In the case that the set of vectors is linearly dependent, express one of them as a linear combination of the other two.

$$\left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -5 \\ -3 \\ -10 \end{bmatrix} \right\}$$

4. (a) (15 points) Find a set of fundamental solutions to the equation

$$\mathbf{x}'(t) = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix} \mathbf{x}(t). \quad (3)$$

- (b) (10 points) Graph in a phase diagram the trajectories of the fundamental solutions found in part (4a). Furthermore, do a *qualitative* sketch of the trajectories of several linear combinations of these fundamental solutions.
- (c) (10 points) Find the solution to the initial value problem given by Eq. (3) and the initial condition

$$\mathbf{x}(0) = \begin{bmatrix} -1 \\ -21 \end{bmatrix}$$