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TA: $\qquad$ Section Time:

Math 20D.
Exam 1
April 23, 2008
No calculators or any other devices are allowed on this exam.
Read each question carefully. If any question is not clear, ask for clarification.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
Answer each question completely, and show all your work.

1. (20 points) Find the solution $y(t)$ to the initial value problem

$$
t y^{\prime}+(1+t) y=2, \quad y(1)=0
$$

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2. (a) (15 points) Compute an implicit expression for the solution $y(t)$ to the initial value problem

$$
y^{\prime}=\frac{e^{-2 t}-e^{2 t}}{3+4 y}, \quad y(0)=0
$$

(b) (5 points) Find the explicit expression for the solution found in part (2a).
3. (30 points) Find all solutions $y(x)$ of the differential equation

$$
\left(\frac{3 y^{3}}{x^{2}}+\frac{5}{x}\right) y^{\prime}+\frac{5 y}{x^{2}}+3 x=0 .
$$

You can leave the solution $y(x)$ expressed in implicit form.
4. (a) (10 points) Find the general solution $y(t)$ of the differential equation:

$$
y^{\prime \prime}-2 y^{\prime}-3 y=0
$$

(b) (10 points) Find the particular solutions $y_{1}(t)$ and $y_{2}(t)$ of the differential equation given in part (4a) corresponding to the initial conditions:

$$
y_{1}(0)=1, \quad y_{1}^{\prime}(0)=0, \quad \text { and } \quad y_{2}(0)=0, \quad y_{2}^{\prime}(0)=1
$$

(c) (10 points) Are the solutions $y_{1}(t)$ and $y_{2}(t)$ found in part (4b) linearly independent or linearly dependent? Justify your answer, and show your work.

