Name: _____ **Sect. Number:** _____

TA: _____ **Sect. Time:** _____

Math 20D.
Quiz 5
May 30, 2008

Answer each question completely, and show your work.
If you use extra paper, write your name on each extra page,
and staple the question page and your own added pages together.

1. (a) (20 points) Find a fundamental matrix $\psi(t)$ for the homogeneous system

$$\boldsymbol{x}'(t) = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix} \boldsymbol{x}(t).$$

   (b) (20 points) Find the solution matrix $\phi(t)$ (which satisfies $\phi(0) = I$) for the system above, and use this matrix $\phi(t)$ to find the solution of the initial value problem

$$\boldsymbol{x}'(t) = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix} \boldsymbol{x}(t), \qquad \boldsymbol{x}(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

2. Consider the inhomogeneous system

$$\boldsymbol{x}'(t) = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix}. \tag{1}$$

   (a) (20 points) Find the general solution of Eq. (1) using the method of undetermined coefficients.

   (b) (20 points) Find the general solution of Eq. (1) using the method of variation of parameters.

   (c) (20 points) Find the general solution of Eq. (1) using that matrix $\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$ is diagonalizable.