Name: ______________________________ **ID Number:** ________________

MTH 340
Exam 3
November 12, 2008

No calculators or any other devices are allowed on this exam.
Read each question carefully. If any question is not clear, ask for clarification.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
Answer each question completely, and show all your work.

1. (25 points) Show that the differential equation below has a regular-singular point at $x_0 = 0$. Propose an appropriate power series expression for the solution, and then find the indicial equation, the recurrence relation and the roots of the indicial equation. Find the first three terms in the power series of the solution; use only the larger root of the indicial equation in case that these roots are different. The differential equation is the following:

$$2x^2\, y'' + 3x\, y' + (2x - 1)\, y = 0.$$

#	Score
1	
2	
3	
4	
Σ	

2. (25 points) Use the Laplace transform to find the solution y of the initial value problem

$$y'' - 2\,y' + 4\,y = \delta(t-3), \qquad y(0) = 1, \qquad y'(0) = -2.$$

3. (25 points) Sketch the graph of the source function g given below, and then use the Laplace transform to find the solution y to the initial value problem

$$y'' - 6\,y = g(t), \qquad y(0) = y'(0) = 0, \qquad g(t) = \begin{cases} 0, & t < \pi, \\ \sin(t-\pi), & t \geqslant \pi. \end{cases}$$

4. (25 points) Find the general solution to the following 2×2 homogeneous linear system,

$$\boldsymbol{x}' = A\boldsymbol{x}, \qquad A = \begin{bmatrix} -3 & \sqrt{2} \\ \sqrt{2} & -2 \end{bmatrix}.$$

Sketch an approximate phase diagram, or phase portrait, of few solutions to the differential equation above.

This is a short list of Laplace transforms and Laplace transform properties that could be useful for the exam. We use the notation $\mathcal{L}[f] = F$.

$$
\begin{aligned}
&f(t) = e^{at} && F(s) = \frac{1}{s-a} && s > \max\{a, 0\},\\
&f(t) = t^n && F(s) = \frac{n!}{s^{(n+1)}} && s > 0,\\
&f(t) = \sin(at) && F(s) = \frac{a}{s^2+a^2} && s > 0,\\
&f(t) = \cos(at) && F(s) = \frac{s}{s^2+a^2} && s > 0,\\
&f(t) = \sinh(at) && F(s) = \frac{a}{s^2-a^2} && s > 0,\\
&f(t) = \cosh(at) && F(s) = \frac{s}{s^2-a^2} && s > 0.
\end{aligned}
$$

The following Laplace transforms could also be useful, where u denotes the step function at $t = 0$, and δ the Dirac delta generalized function:

$$\mathcal{L}[u(t-c)] = \frac{e^{-cs}}{s}, \qquad \mathcal{L}[\delta(t-c)] = e^{-cs}.$$

The following relations could also be useful:

$$\mathcal{L}\big[u(t-c)\, f(t-c)\big] = e^{-cs}\, F(s), \qquad \mathcal{L}\big[e^{ct}\, f(t)\big] = F(s-c),$$

$$\mathcal{L}\big[f^{(n)}(t)\big] = s^n F(s) - s^{(n-1)} f(0) - \cdots - f^{(n-1)}(0).$$