

Name: _____ ID Number: _____

MTH 340

Exam 1

September 12, 2008

No calculators or any other devices are allowed on this exam.

Read each question carefully. If any question is not clear, ask for clarification.

Write your solutions clearly and legibly; no credit will be given for illegible solutions.

Answer each question completely, and show all your work.

1. (a) (20 points) Find the solution to the initial value problem

$$2y' + y = 3t, \quad y(0) = y_0.$$

- (b) (5 points) Find the value of y_0 such that the solution $y(t)$ of the equation above is a linear function of t .

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2. (a) (20 points) Compute an *implicit* expression for the solution $y(x)$ to the initial value problem

$$y' = \frac{x^2}{2y - 4}, \quad y(1) = 0.$$

- (b) (5 points) Find the *explicit* expression for the solution found in part (2a).

3. (25 points) A tank contains a volume $V_0 = 100$ gallons of water with a Q_0 amount of salt dissolved in it. At a time $t_0 = 0$ minutes fresh water is pouring into the tank at a constant rate $r_i = 5$ gallons per minute, while water is also leaving the tank at a constant rate r_o with a salt concentration $q_o(t)$. Consider that there is a mixing mechanism in the tank such that the salt that enters into the tank is *instantaneously mixed* in the tank.

Find the values of the rate r_o and the initial amount of salt Q_0 such that the following two conditions hold: First, the volume of water in the tank remains constant; second, the time needed to reduce the initial amount of salt Q_0 in the tank to the value 10 lb is precisely 20 minutes.

4. (25 points) Show that the following differential equation is exact and then find an implicit expression for all solutions $y(x)$. The differential equation is the following:

$$\sin(xy) y'(x) + xy \cos(xy) y'(x) + e^x + y^2 \cos(xy) = 0.$$