Name: ______________________________ **ID Number:** __________________

MTH Make-up
Exam 1
September 9, 2008

No calculators or any other devices are allowed on this exam.
Read each question carefully. If any question is not clear, ask for clarification.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
Answer each question completely, and show all your work.

1. (25 points) Find the solution $y(t)$ to the initial value problem

$$t\,y' + 2\,y = \sin(t), \qquad t > 0, \qquad y(2\pi) = \frac{1}{2\pi}.$$

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2. (a) (20 points) Compute an *implicit* expression for the solution $y(x)$ to the initial value problem

$$y' = \frac{x(x^2 + e^x)}{4y^3}, \qquad y(0) = -\sqrt{2}.$$

(b) (5 points) Find the *explicit* expression for the solution found in part (2a).

3. (25 points) A tank contains a volume $V_0 = 100$ gallons of water with a Q_0 amount of salt dissolved in it. At a time $t_0 = 0$ minutes fresh water is pouring into the tank at a constant rate r_i, while water is also leaving the tank at a constant rate r_o with a salt concentration $q_o(t)$. Consider that there is a mixing mechanism in the tank such that the salt that enters into the tank is *instantaneously mixed* in the tank.

   Find the values of the rates r_i and r_o such that the following two conditions hold: First, the volume of water in the tank remains constant; second, the time needed to reduce the initial amount of salt Q_0 in the tank to the value $e^{-5}Q_0$ is precisely 25 minutes.

4. (25 points) Show that the following differential equation is exact and then find an implicit expression for all solutions $y(x)$. The differential equation is the following:

$$\left[x^2 + y^2\right]\left[x + y\,y'(x)\right] + 2 = 0.$$