Math 20F.
Midterm Exam 1
October 17, 2005

Read each question carefully, and answer each question completely.
Show all of your work. No credit will be given for unsupported answers.
Write your solutions clearly and legibly. No credit will be given for illegible solutions.

1. (6 points) Consider the system of linear equations

$$
\begin{array}{rcrcrcr}
2x_1 & + & 3x_2 & - & x_3 & = & 6, \\
-x_1 & - & x_2 & + & 2x_3 & = & -2, \\
x_1 & & & + & 2x_3 & = & 2.
\end{array}
$$

(a) Use elementary row operations to write the augmented matrix of the system in echelon form.

(b) Find all solutions of the system. If the system has no solutions, explain how you conclude that.

#	Score
1	
2	
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2. (6 points) Find all the solutions of the non-homogeneous system $A\mathbf{x} = \mathbf{b}$, and write them in parametric form, where

$$A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 1 & 8 \\ 1 & -1 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}.$$

3. (6 points) Consider the matrix A and the vector $\mathbf{b}$ given by

$$A = \begin{bmatrix} 1 & -2 & 7 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}.$$

(a) Is $\mathbf{b}$ in the span of the columns of A? Why?

(b) Are the columns of A linearly independent? Why?

4. (8 points) Let $T : \mathbb{R}^3 \to \mathbb{R}^2 s$ be a linear transformation given by

$$T(\mathbf{e}_1) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad T(\mathbf{e}_2) = \begin{bmatrix} 3 \\ -1 \end{bmatrix}, \quad T(\mathbf{e}_3) = \begin{bmatrix} -2 \\ 2 \end{bmatrix},$$

where

$$\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

(a) Find the matrix A associated to the linear transformation T.

(b) Find $T(-\mathbf{e}_1 + 2\mathbf{e}_2 + 3\mathbf{e}_3)$.

(c) Is T one-to-one? Justify your answer.

(d) Is T onto? Justify your answer.