Math 20F
Quiz 4 (version 2)
May 27, 2005

1. (5.2.18) Find h in the matrix A below such that the eigenspace for $\lambda = 7$ is two-dimensional.

$$A = \begin{bmatrix} 7 & -1 & 3 & -6 \\ 0 & 1 & h & 0 \\ 0 & 0 & 7 & 1 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

The eigenspace of A corresponding to $\lambda = 7$ is equal to $\mathrm{Nul}(A - 7I)$. Thus, we must find the value of h for which dimension of $\mathrm{Nul}(A-7I)$ is 2, which is the value of h for which the number of free variables in the homegeneous system $(A - 7I)\mathbf{x} = \mathbf{0}$ is 2.

$A - 7I = \begin{bmatrix} 0 & -1 & 3 & -6 \\ 0 & -6 & h & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -4 \end{bmatrix}$, and the reduced echelon form of $A - 7I$ is $\begin{bmatrix} 0 & 1 & -3 & 0 \\ 0 & 0 & 18-h & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

Thus, the homogeneous system $(A - 7I)\mathbf{x} = \mathbf{0}$ has 2 free variables and the dimension of the eigenspace of A corresponding to $\lambda = 7$ is 2 when $h = 18$.

2. (5.3.24) A is a 3×3 matrix with two eigenvalues. Each eigenspace is one-dimensional. Is A diagonalizable? Why (or why not)?

Since the dimensions of the eigenspaces of A add up to only 2, A does not have a set of 3 linearly independent eigenvectors; thus, A is not diagonalizable.

3. (6.1.26) Let $\mathbf{u} = \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$, and let W be the set of all $\mathbf{x}$ in $\mathbb{R}^3$ such that $\mathbf{u}\cdot\mathbf{x} = 0$. What theorem (in Chapter 4) can be used to show that W is a subspace of $\mathbb{R}^3$? Describe W in geometric language.

- $\mathbf{u}\cdot\mathbf{x} = \mathbf{u}^T\mathbf{x}$; thus, $W = \mathrm{Nul}\left(\mathbf{u}^T\right)$. By the theorem (Theorem 2 in Chapter 4) stating that the null space of an $m \times n$ matrix A is a subspace of $\mathbb{R}^n$, W is a subspace of $\mathbb{R}^3$ since it is the null space of the 1×3 matrix $\mathbf{u}^T$.
- Geometrically, W is the plane in $\mathbb{R}^3$ orthogonal to the vector $\mathbf{u}$.