Math 20F
Quiz 3 (version 1)
May 13, 2005

1. (3.2.29) Compute $\det B^5$, where $B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$.

$\det B = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 0 \end{vmatrix}$, by subtracting the first column from the third column of B. Thus, $\det B = 1 \cdot \begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} = -2$. Since the determinant is multiplicative, $\det B^5 = (\det B)^5 = (-2)^5 = -32$.

2. (4.4.27) Use coordinate vectors to test the linear independence of the set

$$\left\{1 + t^3,\ 3 + t - 2t^2,\ -t + 3t^2 - t^3\right\}$$

of polynomials. Explain your work.

The standard basis $\mathcal{E}$ for $\mathbb{P}_3$ is $\{1, t, t^2, t^3\}$. Hence,

$$\left[1 + t^3\right]_{\mathcal{E}} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \left[3 + t - 2t^2\right]_{\mathcal{E}} = \begin{bmatrix} 3 \\ 1 \\ -2 \\ 0 \end{bmatrix}, \quad \left[-t + 3t^2 - t^3\right]_{\mathcal{E}} = \begin{bmatrix} 0 \\ -1 \\ 3 \\ -1 \end{bmatrix}$$

The set $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \\ -1 \end{bmatrix} \right\}$ is linearly independent since none of the vectors is a linear combination of the other two, as can be seen by observing the location of the zero entries. Thus, $\{1 + t^3,\ 3 + t - 2t^2,\ -t + 3t^2 - t^3\}$ is linearly independent because the set of corresponding coordinate vectors is linearly independent.

3. (4.6.3) The matrices $A = \begin{bmatrix} 1 & 5 & 2 & 1 & 2 \\ -2 & -10 & 1 & 8 & -9 \\ 3 & 15 & 1 & -7 & 11 \\ 1 & 5 & 1 & -1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 & 0 & -3 & 4 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

are row equivalent.

(a) List rank A and dim Nul A.

rank $A = 2 =$ number of nonzero rows in B, the reduced echelon form of A.

dim Nul $A = 3 =$ number of non-pivot columns in B and A.

(b) Find bases for Col A, Row A, and Nul A.

basis for Col $A = \left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$, the set of pivot columns of A.

basis for Row $A = \{(1, 5, 0, -3, 4),\ (0, 0, 1, 2, -1)\}$, the set of nonzero rows of B.

Since Nul $A = \left\{ \begin{bmatrix} -5x_2 + 3x_4 - 4x_5 \\ x_2 \\ -2x_4 + x_5 \\ x_4 \\ x_5 \end{bmatrix} = x_2 \begin{bmatrix} -5 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 3 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -4 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$,

basis for Nul $A = \left\{ \begin{bmatrix} -5 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$