Print Name: ______________________ **Student Number:** ______________

Section Time: ______________

Math 20F.
Midterm Exam 2
November 21, 2005

Read each question carefully, and answer each question completely.
Show all of your work. No credit will be given for unsupported answers.
Write your solutions clearly and legibly. No credit will be given for illegible solutions.

1. (6 points) Consider the matrices

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}.$$

For each of the following expressions, compute it or explain why it is not defined.

(a) $A + A^T$, and $B + B^T$.

(b) AB and BA.

(c) Find a 2×2 matrix C such that $BC = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$.

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2. (6 points) Find the dimension and a basis for both the null space of A and the column space of A, where

$$A = \begin{bmatrix} 1 & -3 & -8 & -3 \\ -2 & 4 & 6 & 0 \\ 0 & 1 & 5 & 7 \end{bmatrix}.$$

Justify your answers.

3. (6 points) Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3\}$ be two bases of $\mathbb{R}^3$, and suppose that

$$\mathbf{c}_1 = 2\mathbf{b}_1 - \mathbf{b}_2 + \mathbf{b}_3, \quad \mathbf{c}_2 = 3\mathbf{b}_2 + \mathbf{b}_3, \quad \mathbf{c}_3 = -3\mathbf{b}_1 + 2\mathbf{b}_3.$$

(a) Find the change of basis matrix $P_{\mathcal{C}\leftarrow\mathcal{B}}$. Justify your answer.

(b) Consider the vector $\mathbf{x} = \mathbf{c}_1 - 2\mathbf{c}_2 + 2\mathbf{c}_3$. Find $[\mathbf{x}]_{\mathcal{B}}$, that is, the components of $\mathbf{x}$ in the basis $\mathcal{B}$. Justify your answer.

4. (6 points) For which values of the number a are the following matrices invertible? Justify your answer. Find the inverse whenever is possible.

$$A = \begin{bmatrix} 0 & 1 & a \\ 1 & a & 1 \\ -1 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & a & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & a & 1 \\ 1 & 0 & 1 \\ 1 & -a & -1 \end{bmatrix}.$$