Slide 1

Tricks for trigs

- Review: Recursion formulas.
- Integral of some trigonometric functions.
- integrals of $f : I\!R \to \mathbb{C}$.

Slide 2

Reduction formulas are a simple way to write complicated integrals

In the case of the function $\sin(x)$ one has:

$$\int (\sin(x))^n \, dx = -\frac{1}{n} (\sin(x))^{(n-1)} \cos(x) + \frac{(n-1)}{n} \int (\sin(x))^{(n-2)} \, dx.$$

$$\int (\cos(x))^n \, dx = \frac{1}{n} (\cos(x))^{(n-1)} \sin(x) + \frac{(n-1)}{n} \int (\cos(x))^{(n-2)} \, dx.$$

Slide 3

Tricks for sines and cosines

Consider integrals of the form

$$\int [\sin(x)]^m [\cos(x)]^n dx.$$

If m or n is odd, then the integral can be done by *substitution*, recalling:

$$\sin'(x) = \cos(x), \quad \cos'(x) = -\sin(x),$$

$$\sin^2(x) + \cos^2(x) = 1.$$

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Tricks for sines and cosines

Consider integrals of the form

$$\int [\sin(x)]^m [\cos(x)]^n dx.$$

If both m and n are even, then integral above can be done recalling

$$\sin^2(x) = \frac{1}{2}[1 - \cos(2x)],$$

$$\cos^2(x) = \frac{1}{2}[1 + \cos(2x)].$$

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Tricks for tangents and secants

Consider integrals of the form

$$\int [\tan(x)]^n [\sec(x)]^m dx.$$

If n is odd or if m is even, then *substitution* recalling

$$\tan'(x) = \sec^2(x), \quad \sec'(x) = \tan(x)\,\sec(x),$$

$$\sec^2(x) - \tan^2(x) = 1.$$

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More tricks on sines and cosines

The following integrals can be done with the associated formula:

$$\int \sin(mx)\cos(nx)\,dx \quad \to \quad \sin(\alpha)\cos(\beta) = \frac{1}{2}[\sin(\alpha-\beta) + \sin(\alpha+\beta)];$$

$$\int \sin(mx)\sin(nx)\,dx \quad \to \quad \sin(\alpha)\sin(\beta) = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)];$$

$$\int \cos(mx)\cos(nx)\,dx \quad \to \quad \cos(\alpha)\cos(\beta) = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)].$$

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Integration can be generalized to functions $f : \mathbb{R} \to \mathbb{C}$

These functions have the form $f(x) = f_1(x) + i\, f_2(x)$, with $f_1(x)$ and $f_2(x)$ real functions.

Examples:

$$\begin{aligned} f(x) &= 2x + i\, \ln(x), \\ f'(x) &= 2 + i\, \frac{1}{x}, \\ g(x) &= \tan(3x) + i\, e^{2x} \\ g'(x) &= 3\sec^2(3x) + 2i\, e^{2x}. \end{aligned}$$

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Riemann sums, integration, and the FTC can be generalized to function $f : \mathbb{R} \to \mathbb{C}$

$$\int f(x)dx = \int f_1(x)dx + i \int f_2(x)dx.$$

Example:

$$\int e^{iax}dx = \frac{1}{ia}e^{iax} = -\frac{i}{a}e^{iax}.$$

$$\int [x^2 + i\cos(x)]dx = \frac{1}{3}x^3 + i\sin(x).$$

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Complex tricks for sine and cosine

Euler formulas

$$e^{ix} = \cos(x) + i\sin(x),$$
$$e^{-ix} = \cos(x) - i\sin(x),$$

imply that

$$\cos(x) = \frac{1}{2}[e^{ix} + e^{-ix}],$$
$$\sin(x) = \frac{1}{2i}[e^{ix} - e^{-ix}].$$

Therefore, integrals in sines and cosines can be transformed into complex integrals for exponentials.