Slide 1

Miscellaneous topics

- Average of a function.
- Integral form of the Mean Value Theorem.
- Polar coordinates.

Slide 2

Integration provides a way to define the average of a continuous function on a closed interval

Definition 1 *The average of a continuous function $f(x)$ in $[a, b]$ is*

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x)\, dx.$$

Theorem 1 *Let m and M be the minimum and maximum values of a continuous function $f(x)$ in $[a, b]$. Then,*

$$m \leq f_{ave} \leq M.$$

Slide 3

The Mean Value Theorem in differential form

Theorem 2 *If $f(x)$ is differentiable in $[a, b]$ with $f'(x)$ continuous in $[a, b]$, then there exists $c \in (a, b)$ such that*

$$f(b) - f(a) = f'(c)\,(b - a).$$

The Mean Value Theorem in integral form

Theorem 3 *If $f(x)$ is continuous in $[a, b]$, then there exists $c \in (a, b)$ such that*

$$\int_a^b f(x)\,dx = f(c)\,(b - a).$$

Slide 4

Polar coordinates are useful to describe situations with circular symmetry

Definition 2 *Let (x, y) be Cartesian coordinates in $\mathbb{R}^2$. Then, polar coordinates (r, θ) are given by*

$$r = \sqrt{x^2 + y^2}, \quad \theta = \arctan\left(\frac{y}{x}\right).$$

The inverse expression is

$$x = r\cos(\theta), \quad y = r\sin(\theta).$$