Slide 1

Areas and volumes are computed by integration

- Review: Substitution rule.
- Areas between curves (Sec. 6.1).
- Volumes: (Sec. 6.2)
  - General solid.
  - Solids of revolution.

Slide 2

Substitution is an inverse form of the chain rule.

Theorem 1 (Change of variable) *Let $g(x)$ be differentiable in $[a, b]$ with $g'(x)$ continuous in $[a, b]$. Let $f(u)$ be continuous for $u = g(x)$ and $x \in [a, b]$. Then,*

$$\int_a^b f(g(x))\, g'(x)\, dx = \int_{g(a)}^{g(b)} f(u)\, du.$$

Slide 3

Areas between curves are computed with appropriate integrals

Theorem 2 *Let $f(x)$, $g(x)$ be continuous functions in $[a, b]$. If $f(x) \geq g(x)$ with $x \in [a, b]$, then the area between their graphs is*

$$A = \int_a^b [f(x) - g(x)]\, dx.$$

Slide 4

The hypothesis that $f(x) \geq g(x)$ is not needed

Theorem 3 *Let $f(x)$, $g(x)$ be continuous functions in $[a, b]$. Then the area between their graphs is*

$$A = \int_a^b |f(x) - g(x)|\, dx.$$

Just replace $f(x) - g(x)$ by its absolute value $|f(x) - g(x)|$.

Slide 5

Volumes are computed integrating the area function

Theorem 4 *Let S be 3-dimensional body that along the x-axis lies in $[a, b]$. Let $A(x)$ be the cross section area of S perpendicular to the x-axis evaluated at x. Then, the volume of S is*

$$V = \int_a^b A(x)\, dx.$$

Slide 6

The volume of solids of revolution are computed with a particular case of the previous formula

- Given a function $f(x)$ with $x \in [a, b]$, construct a solid of revolution rotating the graph of f around the x-axis.
- The sections perpendicular to the x-axis of such a solid are precisely disks or radius $f(x)$.
- Then, the area of cross sections for such a solid is $A(x) = \pi[f(x)]^2$.

Slide 7

The volume of solids of revolution are computed with a particular case of the previous formula

Theorem 5 *Let S be 3-dimensional solid of revolution obtained rotating the graph of $f(x)$ for $x \in [a, b]$ along the x-axis. Then, the volume of S is*

$$V = \pi \int_a^b [f(x)]^2 \, dx.$$