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Review on Integration (Secs. 5.1 - 5.3)

- Remarks on the course.
- Review: Sec. 5.1-5.3
  - Origins of Calculus.
  - Riemann Sums.
  - New functions from old ones.

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A mathematical description of motion motivated the creation of Calculus

Problem of Motion:

- Given $x(t)$ find $v(t)$ $\leftrightarrow$ Differential Calculus.
- Given $v(t)$ find $x(t)$ $\leftrightarrow$ Integral Calculus.

Derivatives and integrals are operations on functions.

One is the inverse of the other. This is the content of the *Fundamental theorem of Calculus.*

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An integral is a sum of infinite many terms

Definition 1 (Integral of a function) *Let $f(x)$ be a function defined on a interval $x \in [a, b]$. The integral of $f(x)$ in $[a, b]$ is the number given by*

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=0}^{n} f(x_i^*)\,\Delta x,$$

if the limit exists. Given a natural number n we have introduced a partition on $[a, b]$ given by $\Delta x = (b - a)/n$. We denoted $x_i^* = (x_i + x_{i-1})/2$, where $x_i = a + i\Delta x$, $i = 0, 1, \cdots, n$. This choice of the sample point x_i^* is called midpoint rule.

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An integral is a sum of infinite many terms

Continuous functions are integrable. The sum of infinite many terms is finite.

Theorem 1 *If $f(x)$ is continuous in $[a, b]$, then*

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} R_n, \quad \textit{exists.}$$

Notation: $\int_a^b f(x)\,dx$ is called the definite integral of $f(x)$ from a to b.

Notice: $\int_a^b f(x)\,dx$ is a number.

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Properties deduced from the definition

$$\begin{aligned}
\int_a^b f(x)\,dx &= -\int_b^a f(x)\,dx; \\
\int_a^a f(x)\,dx &= 0; \\
\int_a^b c\,dx &= c(b-a); \\
\int_a^b (f \pm g)\,dx &= \int_a^b f\,dx \pm \int_a^b g\,dx;
\end{aligned}$$

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More properties deduced from the definition

$$\begin{aligned}
\int_a^b c\,f(x)\,dx &= c\int_a^b f(x)\,dx; \\
\int_a^b f(x)\,dx &= \int_a^c f(x)\,dx + \int_c^a f(x)\,dx; \\
f \geq 0 &\Rightarrow \int_a^b f\,dx \geq 0; \\
f \geq g &\Rightarrow \int_a^b f\,dx \geq \int_a^b g\,x; \\
m \leq f \leq M &\Rightarrow m(b-a) \leq \int_a^b f\,dx \leq M(b-a).
\end{aligned}$$

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Integration can be used to define new functions from old ones

Theorem 2 *Let $f(x)$ be continuous in $[a,b]$. Then,*

$$F(x) = \int_a^x f(s)\,ds, \quad x \in [a,b],$$

is a continuous functions and $F(a) = 0$.

Examples:

$$\ln(x) = \int_1^x \frac{1}{s}\,ds;$$

$$x^2 = \int_0^x 2s\,ds;$$