Slide 1

The arc length of a curve in space

- Arc length of a curve.
- Arc length function.
- Examples.

Slide 2

The arc length of a curve is a number that measures the extension of the curve

Definition 1 *Let* $\mathbf{r}(t)$ *be a continuously differentiable vector-valued function. The length of the curve associated with* $\mathbf{r}(t)$ *for* $t \in [a, b]$ *is the number given by*

$$\ell_{ba} = \int_a^b |\mathbf{r}'(t)|\, dt.$$

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The arc length of a curve in space has the following form in components

$$\begin{aligned} \mathbf{r}(t) &= \langle x(t), y(t), z(t) \rangle, \\ \mathbf{r}'(t) &= \langle x'(t), y'(t), z'(t) \rangle, \\ \ell_{ba} &= \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\, dt. \end{aligned}$$

Suppose that the curve represents the path traveled by a particle in space. Then, the length of the curve is the integral of the speed, $|\mathbf{v}(t)|$.

So in this case the length of the curve is the distance traveled by the particle.

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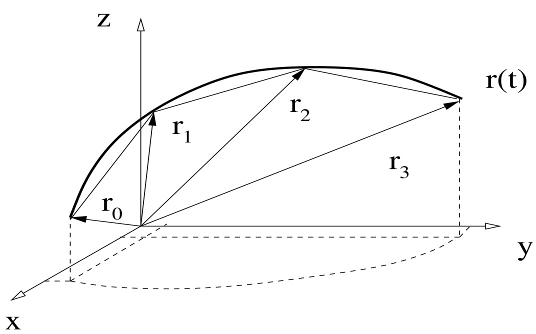
The arc length formula can be obtained as a limit procedure



One adds up the lengths of a polygonal line that approximates the original curve

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The arc length function $\ell(t)$ represents the length up to t of the curve given by $\mathbf{r}(t)$

Definition 2 *Let $\mathbf{r}(t)$ be a continuously differentiable vector-valued function. The arc length function $\ell(t)$ from $t = t_0$ to t is given by*

$$\ell(t) = \int_{t_0}^{t} |\mathbf{r}'(u)| du.$$

$\ell(t)$ **is a scalar function. It satisfies** $\ell(t_0) = 0$

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Reparametrization of a given vector valued function $\mathbf{r}(t)$ using the arc length function

- With $\mathbf{r}(t)$ compute $\ell(t)$, starting at some $t = t_0$.
- Invert the function $\ell(t)$ to find the function $t(\ell)$.
  Example: If $\ell(t) = 3e^{t/2}$, then $t(\ell) = 2\ln(\ell/3)$.
- Compute the composition $\mathbf{r}(\ell) = \mathbf{r}(t(\ell))$.
  That is, replace t by $t(\ell)$.

The function $\mathbf{r}(\ell)$ is the reparametrization of $\mathbf{r}(t)$ using the arc length as the new parameter

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Examples, Sec. 13.4

- (Probl. 14 Sec. 13.4) Find the velocity, acceleration, and speed of the following position function

$$\mathbf{r}(t) = t\sin(t)\mathbf{i} + t\cos(t)\mathbf{j} + t^2\mathbf{k}.$$

- (Probl. 16 Sec. 13.4) Find the velocity and position vectors given the acceleration and initial velocity and position:

$$\mathbf{a}(t) = -10\mathbf{k}, \quad \mathbf{v}(0) = \mathbf{i} + \mathbf{j} - \mathbf{k}, \quad \mathbf{r}(0) = 2\mathbf{i} + 3\mathbf{j}.$$

- Problems with projectiles. Given the initial speed $|\mathbf{v}_0|$ and the initial angle of the projectile with the horizontal, θ, describe the movement of the projectile.