Name: ____________________________ **Section Number:** ______

TA Name: ____________________________ **Section Time:** ______

Math 20C
Midterm Exam 1.
April 28, 2006

No calculators or any other devices are allowed on this exam.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
Read each question carefully. If any question is not clear, ask for clarification.
Answer each question completely, and show all of your work.

1. (a) (5 points) Find all constants c such that the vectors $\mathbf{v} = \langle 2, -c, 3\rangle$ and $\mathbf{w} = \langle c^2, c, -3\rangle$ are perpendicular to each other.

   (b) (5 points) Set $c = 1$ in vectors $\mathbf{v}$ and $\mathbf{w}$ above. In this case, find a unit vector perpendicular to both $\mathbf{v}$ and $\mathbf{w}$.

   (c) (5 points) Keep $c = 1$. Find the scalar projection of $\mathbf{v}$ onto $\mathbf{w}$.

(a)

$$0 = \mathbf{v}\cdot\mathbf{w} = \langle 2, -c, 3\rangle \cdot \langle c^2, c, -3\rangle = 2c^2 - c^2 - 9 = c^2 - 9 \Rightarrow$$
$$\Rightarrow \quad c^2 = 9 \quad \Rightarrow \quad c = \pm 3.$$

(b) $c = 1$ then $\mathbf{v} = \langle 2, -1, 3\rangle$, $\mathbf{w} = \langle 1, 1, -3\rangle$, then

$$\mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 3 \\ 1 & 1 & -3 \end{vmatrix} = \langle (3-3), -(-6-3), (2+1)\rangle \Rightarrow$$

$$\Rightarrow \quad \mathbf{u} = \langle 0, 9, 3\rangle, \quad \Rightarrow |\mathbf{u}| = \sqrt{9^2+9} = \sqrt{9(9+1)} = 3\sqrt{10}.$$

Then a unit vector $\tilde{\mathbf{u}}$ normal to both $\mathbf{v}$ and $\mathbf{w}$ is

$$\tilde{\mathbf{u}} = \frac{\mathbf{u}}{|\mathbf{u}|} = \frac{1}{\sqrt{10}}\langle 0, 3, 1\rangle.$$

(c)

$$P_{\mathbf{v}\text{onto}\mathbf{w}} = \frac{\mathbf{v}\cdot\mathbf{w}}{|\mathbf{w}|} = \frac{2-1-9}{\sqrt{1+1+9}} = -\frac{-8}{\sqrt{11}}.$$

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2. (10 points) Find the equation for the plane that contains the point $P_0 = (3, 2, 1)$ and the line $x = 4 - t$, $y = t$, $z = 3 - 2t$.

The equation of the line in vector form is

$$\mathbf{r}(t) = \langle 4, 0, 3\rangle + \langle -1, 1, -2\rangle t$$

so it tangent vector is $\mathbf{v} = \langle -1, 1, -2\rangle$. The point $P_0 = (3, 2, 1)$ is in the plane. A second point in the plane is any point in the line, for example P_1 corresponding to the head of $\mathbf{r}(t = 0) = \langle 4, 0, 3\rangle$. Then a second vector tangent to the plane is $\overrightarrow{P_0P_1} = \langle 1, -2, 2\rangle$. Then, a normal to the plane is given by

$$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 1 & -2 \\ 1 & -2 & 2 \end{vmatrix} = \langle (2-4), -(-2+2), (2-1)\rangle \Rightarrow$$

$$\Rightarrow \quad \mathbf{n} = \langle -2, 0, -1\rangle.$$

So, the equation of the plane is

$$-2(x - 3) + 0\,(y - 2) - (z - 1) = 0, \quad \Rightarrow \quad 2x - z = 5.$$

3. (a) (10 points) Find the position and velocity vector functions of a particle that moves with an acceleration function $\mathbf{a}(t) = \langle 0, 0, -10 \rangle \; m/sec^2$, knowing that the initial velocity and position are given by, respectively, $\mathbf{v}(0) = \langle 0, 2, 1 \rangle \; m/sec$ and $\mathbf{r}(0) = \langle 0, 0, 2 \rangle \; m$.

(b) (5 points) Draw an approximate picture of the graph of $\mathbf{r}(t)$ for $t \geq 0$.

$$\mathbf{a}(t) = \langle 0, 0, -10 \rangle,$$

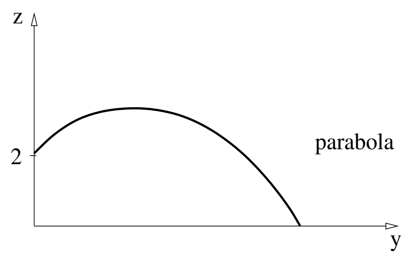
$$\mathbf{v}(t) = \langle v_{0x}, v_{0y}, -10\,t + v_{0z} \rangle, \quad \mathbf{v}(0) = \langle 0, 2, 1 \rangle \quad \Rightarrow \begin{cases} v_{0x} = 0, \\ v_{0y} = 2, \\ v_{0z} = 1. \end{cases}$$

$$\mathbf{v}(t) = \langle 0, 2, -10\,t + 1 \rangle.$$

$$\mathbf{r}(t) = \langle r_{0x}, 2t + r_{0y}, -5t^2 + t + r_{0z} \rangle, \quad \mathbf{v}(0) = \langle 0, 0, 2 \rangle \quad \Rightarrow \begin{cases} r_{0x} = 0, \\ r_{0y} = 0, \\ r_{0z} = 2. \end{cases}$$

$$\mathbf{r}(t) = \langle 0, 2t, -5t^2 + t + 2 \rangle.$$

(b)



4. (10 points) Reparametrize the curve $\mathbf{r}(t) = \left\langle 2\sin(t^2), \frac{3}{2}t^2, 2\cos(t^2) \right\rangle$ with respect to its arc length measured from $t = 1$ in the direction of increasing t.

(Just in case you read it too fast, we repeat: starting at $t = 1$.)

$$\mathbf{r}'(t) = \langle 4t\cos(t^2), 3t, -4\sin(t^2) \rangle,$$

$$\begin{aligned} |\mathbf{r}'(t)| &= \sqrt{16t^2\cos^2(t^2) + 9t^2 + 16\sin^2(t^2)}, \\ &= \sqrt{16t^2 + 9t^2}, \\ &= \sqrt{16+9}t, \\ &= 5t. \end{aligned}$$

$$s = \int_1^t 5\tilde{t}\, d\tilde{t} = \frac{5}{2}\left(\tilde{t}^2\Big|_1^t \right) = \frac{5}{2}(t^2 - 1).$$

$$t^2 = \frac{2}{5}s + 1.$$

$$\mathbf{r}(s) = \left\langle 2\sin\left(\frac{2}{5}s + 1\right), \frac{3}{2}\left(\frac{2}{5}s + 1\right), 2\cos\left(\frac{2}{5}s + 1\right) \right\rangle.$$