Print Name: ______________________ **Student Number:** ______________

Math 20C.
Midterm Exam 1
October 17, 2005

Read each question carefully, and answer each question completely.
Show all of your work. No credit will be given for unsupported answers.
Write your solutions clearly and legibly. No credit will be given for illegible solutions.

1. (6 points) Consider the vectors $\mathbf{v} = \langle 6, 2, -3 \rangle$ and $\mathbf{w} = \langle -2, 2, 1 \rangle$.

   (a) Find a vector normal to both, $\mathbf{v}$ and $\mathbf{w}$.

$$\mathbf{v} \times \mathbf{w} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 2 & -3 \\ -2 & 2 & 1 \end{vmatrix} = (2+6)\mathbf{i} - (6-6)\mathbf{j} + (12+4)\mathbf{k} = \langle 8, 0, 16 \rangle.$$

   A solution is any vector proportional to $\mathbf{v} \times \mathbf{w}$. For example $\mathbf{u} = \langle 1, 0, 2 \rangle$.

   (b) Find the area of the parallelogram formed by $\mathbf{v}$ and $\mathbf{w}$.

$$A = |\mathbf{v} \times \mathbf{w}| = \sqrt{8^2 + 16^2} = \sqrt{8^2(1+4)} = 8\sqrt{5}.$$

   (c) Find a vector of length one in the direction of $\mathbf{v}$.

$$|\mathbf{v}| = \sqrt{36 + 4 + 9} = \sqrt{49} = 7.$$

$$\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{1}{7}\langle 6, 2, -3 \rangle.$$

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2. (6 points) Find an equation for the plane that passes through the points $(1, 1, 1)$, $(1, -1, 1)$, and $(0, 0, 2)$.

Let
$$P = (1, 1, 1), \quad Q = (1, -1, 1), \quad R = (0, 0, 2).$$
Then,
$$\vec{PQ} = \langle 0, -2, 0 \rangle, \quad \vec{PR} = \langle -1, -1, 1 \rangle,$$
$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -2 & 0 \\ -1 & -1 & 1 \end{vmatrix} = (-2 - 0)\mathbf{i} - (0 - 0)\mathbf{j} + (0 - 2)\mathbf{k} = \langle -2, -0, -2 \rangle.$$
Take $\mathbf{n} = \langle 2, 0, 2 \rangle$, and a point $R = (0, 0, 2)$. Then, the equation of the plane is
$$2(x - 0) + 0(y - 0) + 2(z - 2) = 0,$$
$$x + z = 2.$$

3. (6 points) Consider the line given by $\mathbf{r}(t) = \langle 0, 1, 1\rangle + \langle 1, 2, 3\rangle t$ and the plane given by $2x + y - z = 1$.

   (a) Does the line intersect the plane? If yes, find the intersection point. In any case, justify your answer.

   The line

$$x = t, \quad y = 1 + 2t, \quad z = 1 + 3t,$$

   intersect the plane $2x + y - z = 1$ if there is a solution t for the equation

$$2t + (1 + 2t) - (1 + 3t) = 1, \Rightarrow t = 1.$$

   Therefore, the point of intersection has coordinates $x = 1$, $y = 3$, $z = 4$, then

$$P = (1, 3, 4).$$

   (b) Find the equation of the line, passing through the point $(0, 1, 1)$ and orthogonal to the plane given above.

The tangent to the line is the normal to the plane, so the result is

$$\mathbf{r}(t) = \langle 0, 1, 1\rangle + \langle 2, 1, -1\rangle t.$$

4. (6 points) A particle moves in a plane with a velocity function given by the expression $\mathbf{v}(t) = \langle 2\sin(t), 3\cos(t) \rangle$, for $t \geq 0$.

(a) Find the acceleration $\mathbf{a}(t)$ function of the particle.

$$\mathbf{a}(t) = \mathbf{v}'(t) = \langle 2\cos(t), -3\sin(t) \rangle.$$

(b) Find the position function $\mathbf{r}(t)$ of the particle knowing that the initial position of the particle is $\mathbf{r}(0) = \langle -1, 1 \rangle$.

$$\mathbf{r}(t) = \langle -2\cos(t) + r_{0x}, 3\sin(t) + r_{0y} \rangle.$$

$$\mathbf{r}(0) = \langle -2 + r_{0x}, r_{0y} \rangle = \langle -1, 1 \rangle, \Rightarrow r_{0x} = 1, \quad r_{0y} = 1.$$

Then, the position function is

$$\mathbf{r}(t) = \langle -2\cos(t) + 1, 3\sin(t) + 1 \rangle.$$