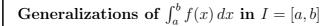


Slide 1

- Review: Improper integrals type I.
- Type II: Three main possibilities.
- Limit of an infinite sequence.



Integrals on infinite domains are called improper integrals of type I

Slide 2

• Type I: The interval is infinite: $I = (-\infty, b]$, or $I = [a, \infty)$ or $I = (-\infty, \infty)$.

Integrals of divergent functions on finite domains are called improper integrals of type II.

• Type II: f(x) is not bounded at one or more points in [a, b]. (f(x) can have a vertical asymptote in [a, b].)

Type II: Vertical asymptote at b

Possibility (a):

Definition 1 If f(x) is continuous in [a, b) then

Slide 3

$$\int_{a}^{b^{-}} f(t) \, dt = \lim_{x \to b^{-}} \int_{a}^{x} f(t) \, dt.$$

The integral is said to converge if the limit exists and it is finite.

Otherwise the integral is said to diverge.

Type II: Vertical asymptote at a

Possibility (b):

Definition 2 If f(x) is continuous in (a, b] then

Slide 4

$$\int_{a^+}^{b} f(t) \, dt = \lim_{x \to a^+} \int_{x}^{b} f(t) \, dt.$$

The integral is said to converge if the limit exists and it is finite.

Otherwise the integral is said to diverge.

Slide 5

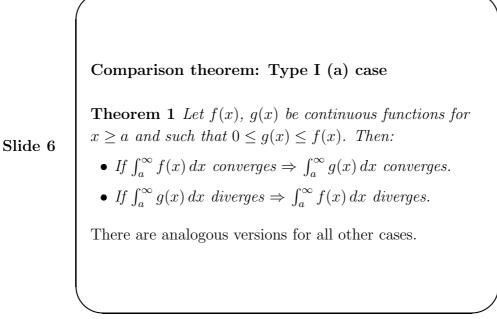
Type II: Vertical asymptote in the interior

Possibility (c):

Definition 3 If f(x) has a vertical asymptote at $c \in (a, b)$, then

$$\int_{a}^{b} f(t) dt = \int_{a}^{c^{-}} f(t) dt + \int_{c^{+}}^{b} f(t) dt$$

provided that both integrals in the right hand side are convergent.



A sequence is a function whose domain are the positive integers

Definition 4 If for every positive integer n there is associated a real number a_n , then the ordered set

Slide 7

 $a_1, a_2, \cdots, a_n, \cdots$

is called an infinite sequence. Is is denoted as $\{a_n\}$.

Definition 5 A function $f : \mathbb{Z}^+ \to \mathbb{R}$ is called an infinite sequence.

The limits of sequences is the same as in functions of real numbers

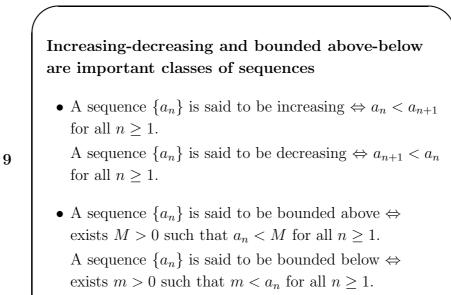
Definition 6 The sequence $\{a_n\}$ is said to have the limit L is for all $\epsilon > 0$ there exists a number N > 0 such that

Slide 8

$$|a_n - L| < \epsilon$$
, for all $n \ge N$.

In this case we say $\lim_{n\to\infty} a_n = n$ or $a_n \to L$ as $n \to \infty$. We say that the sequence converges.

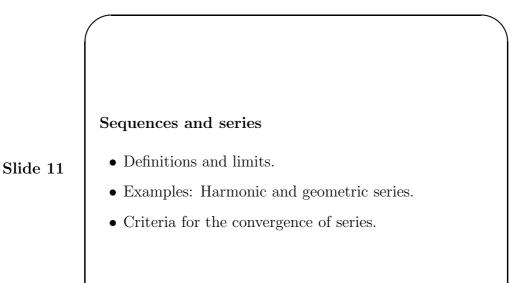
Otherwise, we say that the sequence diverges.



Important tool to show that a sequence converges
If {a_n} is increasing and bounded above then converges.
If {a_n} is decreasing and bounded below then converges.

Slide 9

Slide 10



A sequence is a function whose domain are the positive integers

Definition 7 A function $f : \mathbb{Z}^+ \to \mathbb{R}$ is called an infinite sequence.

Slide 12

Notation: f(n), or f_n , or a_n , or $\{a_n\}$.

Then, for every positive integer n there is associated a real number a_n , and so the ordered set

 $a_1, a_2, \cdots, a_n, \cdots$

is called an infinite sequence.

Here are some of the most popular sequences • Harmonic sequence: $\left\{\frac{1}{n}\right\} = \left\{1, \frac{1}{2}, \frac{1}{3}, \cdots\right\}.$ • Geometric sequence: $\left\{\frac{1}{2^{(n-1)}}\right\} = \left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \cdots\right\}.$ • $\left\{\left(1 + \frac{a}{n}\right)^n\right\}.$

Slide 13

The limits of sequences is the same as in functions of real numbers

Definition 8 The sequence $\{a_n\}$ is said to have the limit L is for all $\epsilon > 0$ there exists a number N > 0 such that

Slide 14

$$|a_n - L| < \epsilon$$
, for all $n \ge N$.

In this case we say that the sequence is convergent and $\lim_{n\to\infty} a_n = n \text{ or } a_n \to L \text{ as } n \to \infty.$

Otherwise, we say that the sequence diverges.

Here are the limits of some of the most popular sequences

• Harmonic sequence:

$$\lim_{n \to \infty} \frac{1}{n} = 0.$$

Slide 15

• Geometric sequence:

$$\lim_{n \to \infty} \frac{1}{2^{(n-1)}} = 0$$
•
$$\lim_{n \to \infty} \left(1 + \frac{a}{n}\right)^n = e^a.$$

A series is a sequence of partial sums Definition 9 A series is a sequence $\{s_n\}$ where s_n has the form $s_n = \sum_{k=1}^n a_k$.

Given $a_1, a_2, a_3, \dots, a_n, \dots$, form the partial sums

Slide 16

 $s_1 = a_1, \quad s_2 = a_1 + a_2, \quad s_3 = a_1 + a_2 + a_3,$

$$s_n = a_1 + \dots + a_n = \sum_{k=1}^n a_k.$$

Then, $\{s_n\}$ or equivalently is a series.

Here are some of the most popular series • Harmonic series: $\sum_{k=1}^{n} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}.$ • Geometric series: $\sum_{k=1}^{n} \frac{1}{2^{(k-1)}} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^{(n-1)}}.$

Slide 17

Limits of series involves adding infinitely many terms

Definition 10 The sum $\sum_{k=1}^{\infty} a_k$ is convergent and has the value s if the sequence of partial sums converges to s, that is,

$$\sum_{k=1}^{\infty} a_k = \lim_{n \to \infty} \sum_{k=1}^n a_n = s.$$

If the series $\{\sum_{k=1}^{n} a_k\}$ is not convergent, we call it divergent.

Here are the sum of some of the most popular series

• Harmonic series:

$$\sum_{k=1}^{\infty} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{4} + \dots \quad \text{diverges.}$$

• Geometric series:

$$\sum_{k=1}^{\infty} \frac{1}{2^{(k-1)}} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = 2.$$

This is a very rough criteria to check whether a series converges or not
First a result:
Slide 20
Theorem 2 If
$$\sum_{n=1}^{\infty} a_n$$
 is convergent, then
 $\lim_{n\to\infty} a_n = 0$.
Then the criteria:
Theorem 3 If $\lim_{n\to\infty} a_n$ does not exist or if
 $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ is divergent.

Slide 19