Name: ______________________ **PID:** ____________

TA: __________________ **Sec. No:** ____ **Sec. Time:** ____

Math 20B.
Final Examination
March 22, 2006

Turn off and put away your cell phone.
No calculators or any other devices are allowed on this exam.
You may use one page of notes, but no books or other assistance on this exam.
Read each question carefully, answer each question completely, and show all of your work.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

1. (6 points) Find a function $y = y(t)$ that satisfies the following differential equation subject to the given initial condition.

$$\begin{aligned} y' &= 4te^{-y} \\ y(1) &= 0 \end{aligned}$$

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2. (12 points) The integral of $\sin^2(x)$ may be found using several different methods.

(a) Find $\int \sin^2(x)\,dx$ using the double-angle identity for $\cos(2x)$.

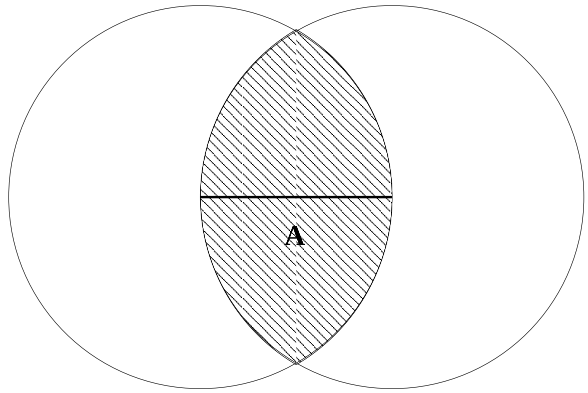
(b) Find $\int \sin^2(x)\,dx$ using integration by parts.

2. (c) Find $\int \sin^2(x)\,dx$ by first writing $\sin^2(x)$ in terms of the complex exponential. Leave the result in exponential form.

3. (6 points) Evaluate the integral

$$\int \frac{2x^2 + 5x - 17}{x^2 + 2x - 15}\, dx$$

4. (6 points) Find the volume common to two spheres, each with radius A, if the center of each sphere lies on the surface of the other.



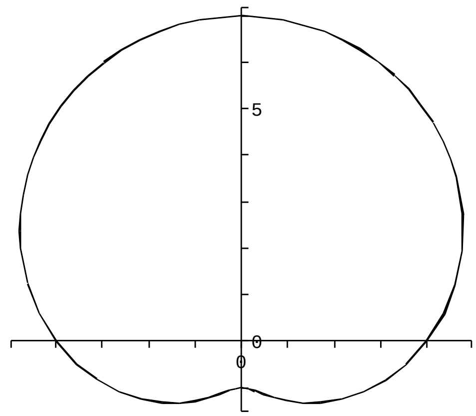
5. (6 points) The function $(x-1)^2 \ln(x)$ is represented as a Taylor series about $x = 1$ by

$$\sum_{n=3}^{\infty} \frac{(-1)^{n+1}}{n-2}(x-1)^n.$$

Find the radius of convergence for this series.

6. (6 points) Find the area enclosed by the polar curve

$$r = 4 + 3\sin(\theta).$$



7. (6 points) The Taylor "series" representing a polynomial is a polynomial. For example, x^2 is represented as a Taylor polynomial about $x = 1$ by $1 + 2(x - 1) + (x - 1)^2$. Represent the polynomial

$$x^3 - 4x^2 + 4x + 1$$

as a Taylor polynomial about $x = 2$.

8. (8 points) z is a complex number written in polar form as

$$z = e^{i\frac{\pi}{3}}.$$

(a) Write z in rectangular form, $a + bi$.

(b) Write z^{11} in rectangular form, $a + bi$.

(c) Find the 4^{th} roots of z in polar form, $r\,e^{i\theta}$.