

Name: \_\_\_\_\_ Section Number: \_\_\_\_\_

TA Name: \_\_\_\_\_ Section Time: \_\_\_\_\_

**Math 20B.**  
**Final Examination**  
**March 16, 2005**

*You may use one page of notes, but no other assistance on this exam.*

*Read each question carefully, answer each question completely, and show all of your work.*

*Write your solutions clearly and legibly; no credit will be given for illegible solutions.*

*If any question is not clear, ask for clarification.*

1. (6 points)

(a) Evaluate  $\int_1^x (5 + t)e^{-t} dt$ .

(b) Determine if  $\int_1^{\infty} (5 + t)e^{-t} dt$  converges; if it does, determine its value.

#	Score
1	
2	
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$\Sigma$	

2. (5 points) Clearly circle the letter of the direction field for each of the following differential equations. Only one answer will be accepted per question. (Yes, there is an extra direction field.)

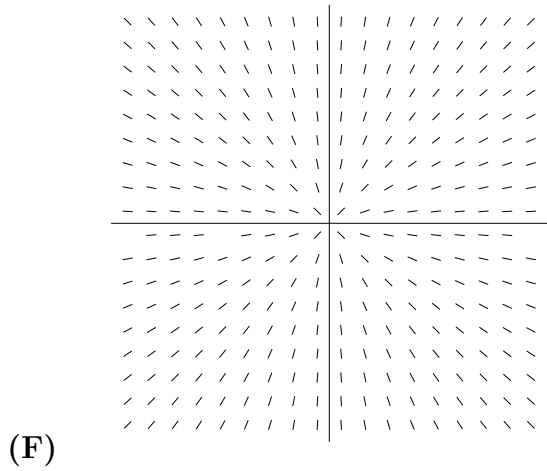
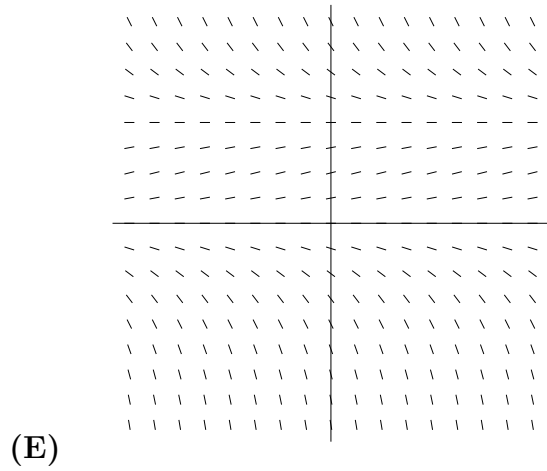
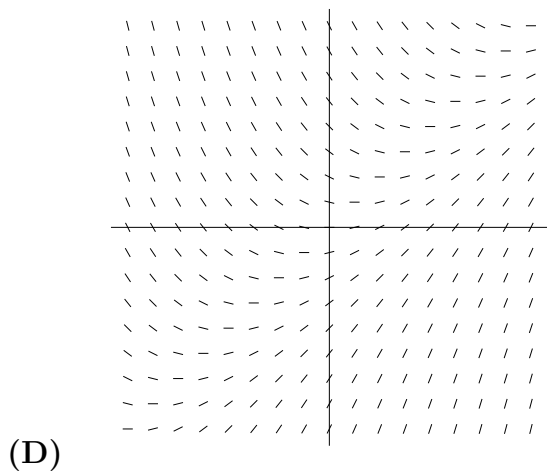
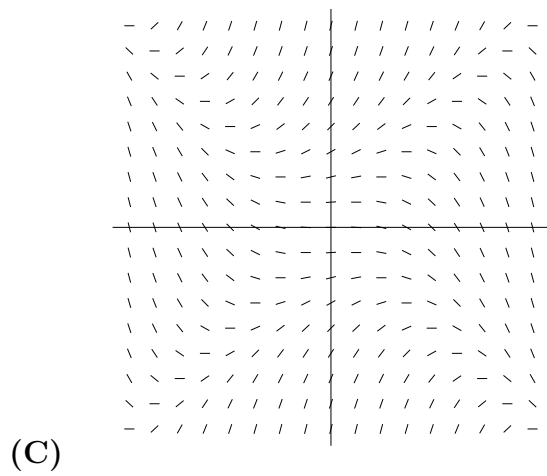
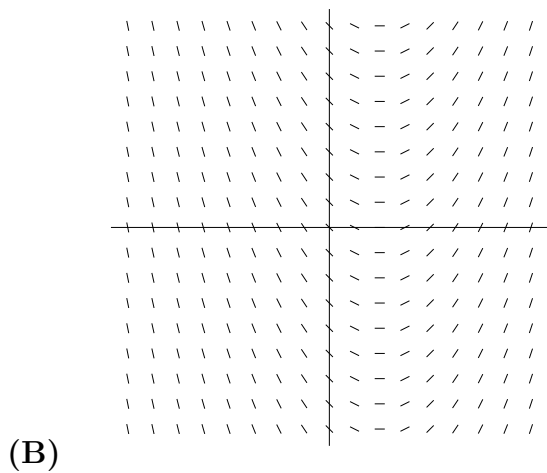
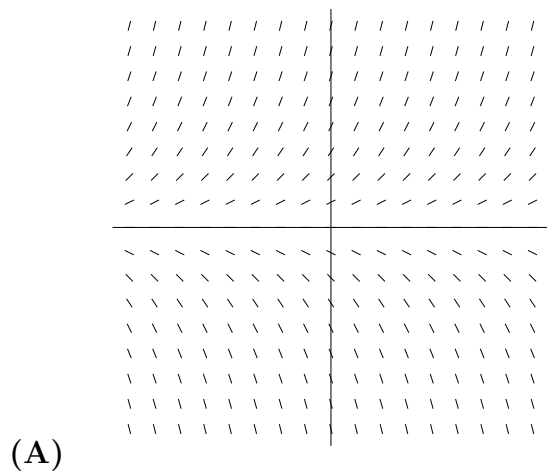
(i)  $y' = x - y$       A              B              C              D              E              F

(ii)  $y' = 2y$               A              B              C              D              E              F

(iii)  $y' = y(1 - y)$       A              B              C              D              E              F

(iv)  $y' = 2x - 1$         A              B              C              D              E              F

(v)  $y' = \frac{y}{x}$               A              B              C              D              E              F



3. (6 points) Evaluate the indefinite integral

$$\int \frac{6x^2 - 3x + 1}{(4x + 1)(x^2 + 1)} dx.$$

4. (4 points) Evaluate the definite integral

$$\int_0^{\frac{\pi}{2}} \cos(x) \sin^{11}(x) dx$$

5. (4 points) Evaluate the indefinite integral

$$\int i (e^{5ix} - e^{-5ix}) dx.$$

Write the result using only real-valued functions; the resulting expression should not contain the imaginary number  $i$ .

6. (6 points) A region in the plane is bounded by the  $x$ -axis and the curve  $y = 3 \cos(x)$  with  $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ .

(a) Find the area of the region.

(b) Find the  $y$ -coordinate of the centroid of the region.

7. (4 points) The base of a solid  $S$  lies in the  $xy$ -plane and is bounded by the  $x$ -axis, the  $y$ -axis, and the parabola  $y = 1 - \frac{x^2}{9}$ . Cross sections of the solid  $S$  perpendicular to the  $x$ -axis are squares. Find the volume of  $S$ .

8. (4 points) Find the solution to the following initial value problem.

$$\begin{cases} y' = yx + 2x \\ y(0) = 1 \end{cases}$$



9. (4 points) Find the area enclosed by the polar curve  $r(\theta) = \sqrt{3 \sin(\theta)}$ .

